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Optics: It is the branch of physics which deals with the study of light and its instruments.

Refraction of light: The process of bending of light when it travels from one medium to another medium is called refraction of light.

When light moves from one medium to another medium then velocity of light changes due to changes in density. As a result, light bend.

Refraction of light on glass slab.

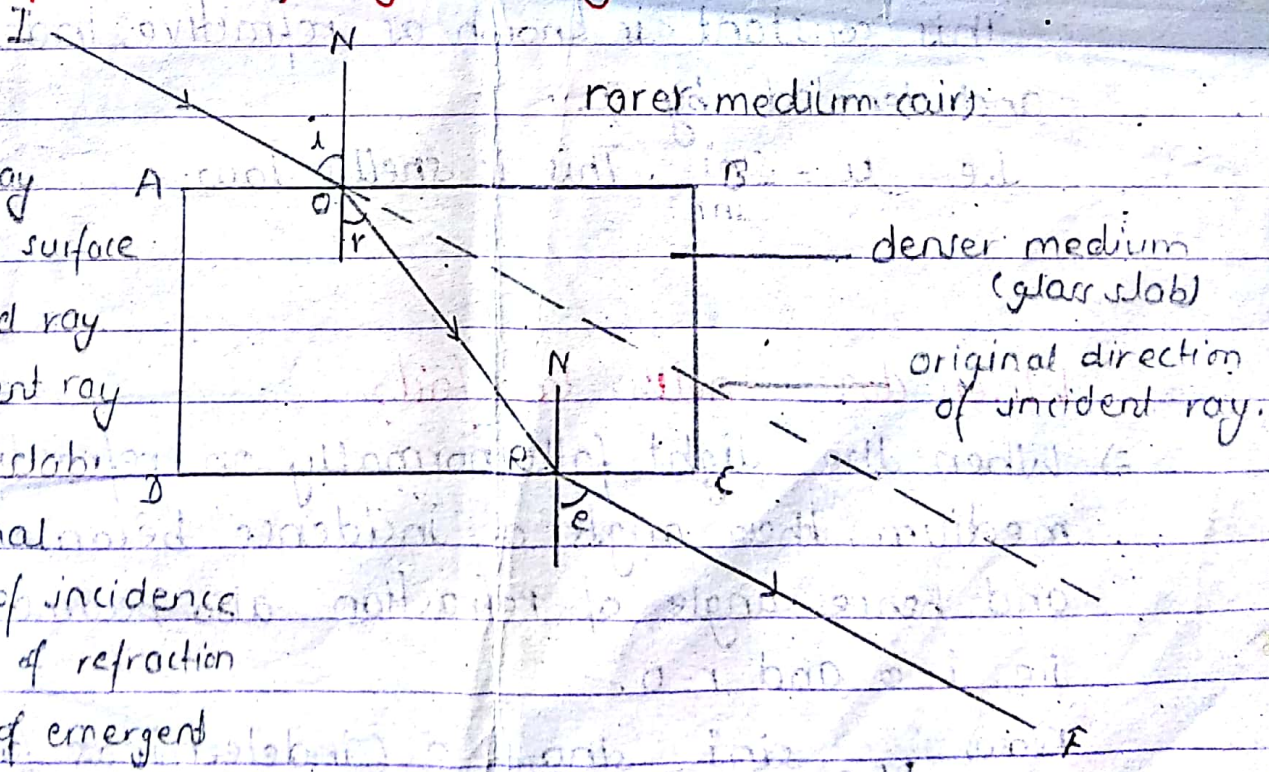


Fig: Refraction of light on glass slab

(2)

laws of refraction:

The laws of refraction are given belows:

- (i) The incident ray, refracted ray and normal all lie at the point on the same plane.
- (ii) When light travels from rarer medium to denser medium it bends towards normal and when it travels from denser to rarer medium it bends away from normal.
- (iii) For any two medium, the ratio of sine of angle of incidence to the sine of angle of refraction is always constant that is (i.e) $\frac{\sin i}{\sin r} = \text{constant}$.

This constant is known as refractive index and denoted by μ .

i.e $\mu = \frac{\sin i}{\sin r}$. This is snell's law.

When does snell's law fail?

- ⇒ When the light falls normally on refracting medium, then angle of incidence become 0 and hence angle of refraction also become 0.
i.e $i=0$ and $r=0$.

Now, $\mu = \frac{\sin i}{\sin r} = \frac{\sin 0}{\sin 0} = \frac{0}{0}$ (indeterminate form)

As a result, snell's law fails.

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Absolute refractive index:

The ratio of velocity of light in vacuum to the velocity of light in ~~ray~~ any medium is known as absolute refractive index.

It is denoted by μ_a .

$\therefore$ Absolute refractive index = $\frac{\text{velocity of light in vacuum}}{\text{velocity of light in any medium}}$

$$\mu_a = \frac{c}{v}$$

It has no unit and dimension. It is a pure number.

Principle of reversibility of light:

Statement: If light after suffering any number of refraction has its final path reversed, it travel back along the same path in the opposite direction.

Explanation:

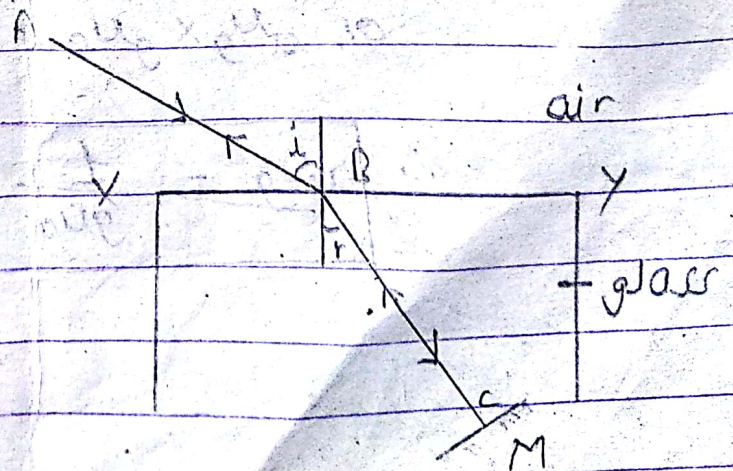


Fig. Principle of reversibility.

(X)
Let us consider a light AB incident on air glass surface XY and refracted along BC as shown in figure.

Here, $\angle i$ = angle of incidence.

$\angle r$ = angle of refraction.

From Snell's law

$$\mu_g = \frac{\sin i}{\sin r} \quad \text{--- (i)}$$

If we place mirror M perpendicular to the direction of refracted ray then light is refracted along BA as shown in figure.

From Snell's law,

$$\mu_a = \frac{\sin r}{\sin i} \quad \text{--- (ii)}$$

Multiplying eq (i) and (ii), we get,

$$\mu_g \times \mu_a = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i}$$

$$\text{or, } \mu_g \times \mu_a = 1$$

$$\therefore \mu_g = \frac{1}{\mu_a}$$

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Refraction through a compound plate.

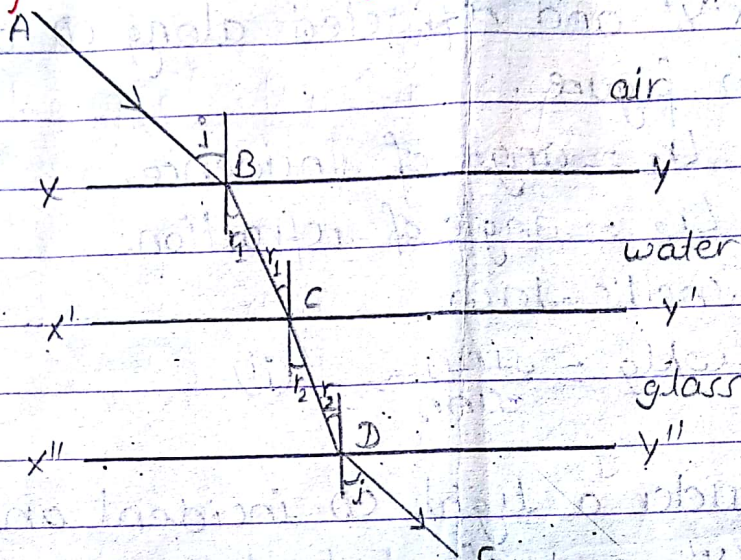


Fig. Refraction through compound plate.

A compound plate consists of three media - air, water and glass. Let XY be a surface of separation between air and water, $X'Y'$ be a surface of separation between water and glass, and $X''Y''$ be a surface of separation between glass and air, let

let us consider a light AB incident on surface XY and refracted along BC as shown in figure.

Here, i = angle of incidence

r_1 = angle of refraction

From Snell's law,

$$a_{w/a} = \frac{\sin i}{\sin r_1} \quad \text{--- (1)}$$

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Let us consider a light BC incident on surface $x'y'$ and refracted along CD as shown in figure.

Here, i_1 = angle of incidence.

r_1 = angle of refraction.

From Snell's Law,

$$\omega\mu_g = \frac{\sin i_1}{\sin r_1} \quad \text{--- (i)}$$

Let us consider a light CD incident on surface $x''y''$ and refracted along DE as shown in figure.

Here, i_2 = angle of incidence.

r_2 = angle of refraction.

From Snell's law,

Now,
$$g\mu_o = \frac{\sin r_2}{\sin i_2} \quad \text{--- eqn (ii)}$$

Multiplying eqn (i), (ii) and (iii).

$$a\mu_o \times \omega\mu_g \times g\mu_o = \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_2}$$

$$\text{or, } a\mu_o \times \omega\mu_g \times g\mu_o = 1$$

$$\text{or, } a\mu_o \times \omega\mu_g = \frac{1}{g\mu_o}$$

$$\text{or, } a\mu_o \times \omega\mu_g = a\mu_g \quad [\because a\mu_g \cdot g\mu_o = 1]$$

$$\therefore \boxed{\omega\mu_g = \frac{a\mu_g}{a\mu_o}}$$

(A)

Lateral shift:

When a ray of light incident on glass slab then emergent ray coming out of the slab will be parallel to initial direction of incident ray. The perpendicular distance between emergent ray and initial direction of incident ray, is called lateral shift. It is denoted by δ *delta*.

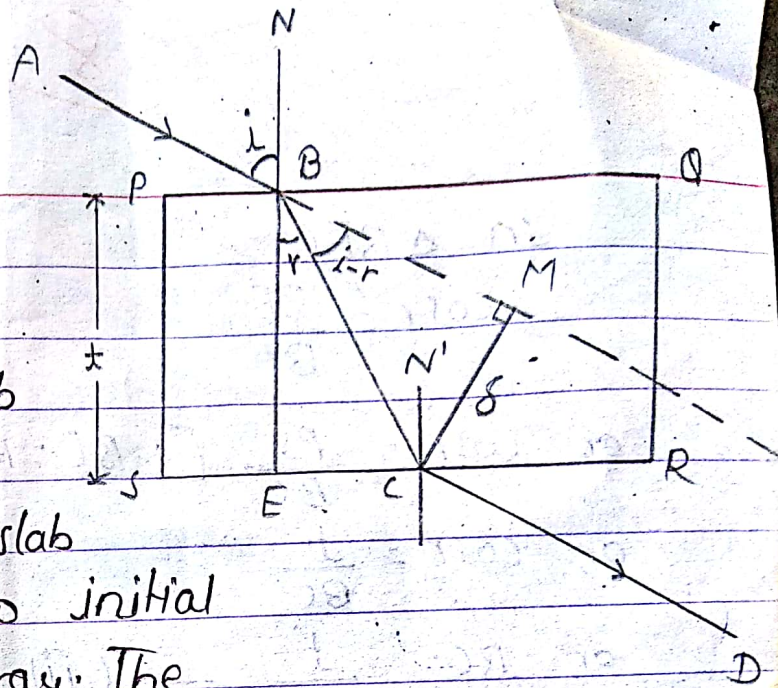


Fig: Lateral shift

Let us consider a ray of light AB incident on glass slab PQRS having thickness t . It is refracted along BC and finally emerges along CD as shown in figure.

Let, i = angle of incidence
 r = angle of refraction

Here, $CM = \delta$ is lateral shift.

In $\triangle BCM$,

$$\sin(i-r) = \frac{CM}{BC}$$

$$\text{or, } CM = BC \sin(i-r)$$

$$\text{or, } \delta = BC \sin(i-r) \quad \text{--- (i)}$$

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In $\triangle BEC$,

$$\cos r = \frac{BE}{BC}$$

$$\text{or, } \cos r = \frac{PS}{BC} \quad [\because BE = PS]$$

$$\text{or, } \cos r = \frac{t}{BC}$$

$$\text{or, } BC = \frac{t}{\cos r} \quad \text{--- (ii)}$$

From equation (i) and (ii), we get

$$\delta = BC \sin(i-r)$$
$$\therefore \boxed{\delta = \frac{t \sin(i-r)}{\cos r}}$$

This shows that lateral shift increases with increase in angle of incidence.

Special cases:

(i) When $i = 0$, then

$$\delta = \frac{t \sin(i-r)}{\cos r}$$

$$\text{or, } \delta = \frac{t \sin(0-0)}{\cos 0} \quad [i=0, \text{ then } r=0]$$

$$\therefore \boxed{\delta = 0}$$

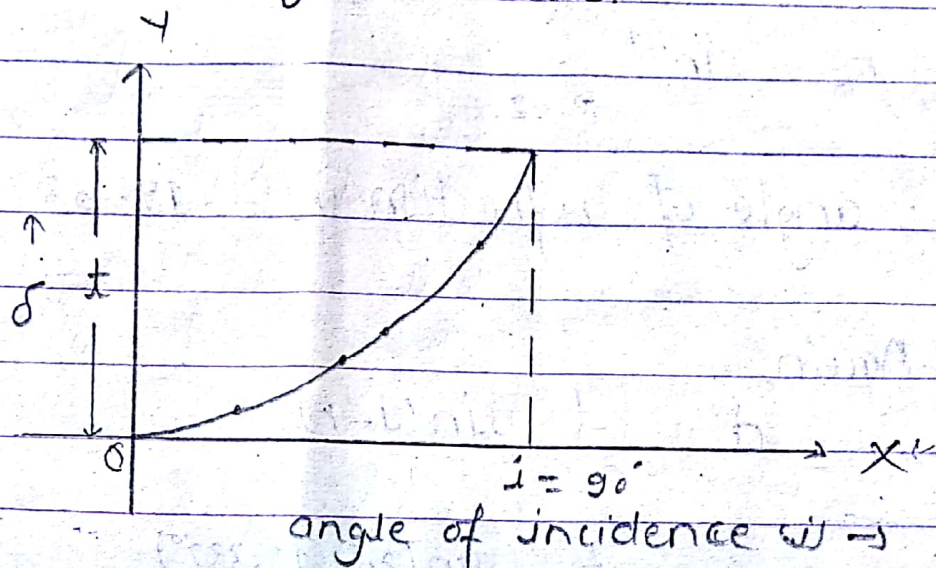
(ii) When $i = 90^\circ$, then

$$\delta = \frac{t \sin(90-r)}{\cos r}$$

$$\text{or, } \delta = \frac{t \cos r}{\cos r} \quad \therefore \boxed{\delta = t}$$

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The variation of lateral shift with an angle of incidence is given below:



Q. A ray of light incident on glass slab having thickness 3 cm by making an angle of 30° with glass slab. Find the value of lateral shift if refractive index of glass is 1.51?

Soln Here given,

Thickness of glass slab (t) = 3 cm

angle of incidence (i) = 30°

Refractive index of glass (μ_g) = 1.51

Value of lateral shift (δ) = ?

Now,

from snell's law,

$$\mu_g = \frac{\sin i}{\sin r}$$

$$\text{or, } 1.51 = \frac{\sin 30^\circ}{\sin r}$$

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$$\text{or, } \sin r = \frac{1/2}{1.51}$$

$$\text{or, } r = \sin^{-1} \frac{1}{3.02}$$

$\therefore$ angle of refraction (r) = 19.337°

Again,

$$\delta = \frac{t \sin(i-r)}{\cos r}$$

$$= \frac{t \sin(30^\circ - 19.337^\circ)}{\cos 19.337^\circ}$$

$$= \frac{3 \sin 10.663^\circ}{0.944}$$

$$= \frac{3 \times 0.185}{0.944}$$

$$= 0.588 \text{ cm}$$

Hence, value of lateral shift is 0.588 cm .

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Real and Apparent depth:

The actual depth of position of object from the interface of rarer and denser medium is known as real depth. And the depth which can be seen from the rarer medium is apparent depth.

Let

i = angle of incidence

r = angle of refraction

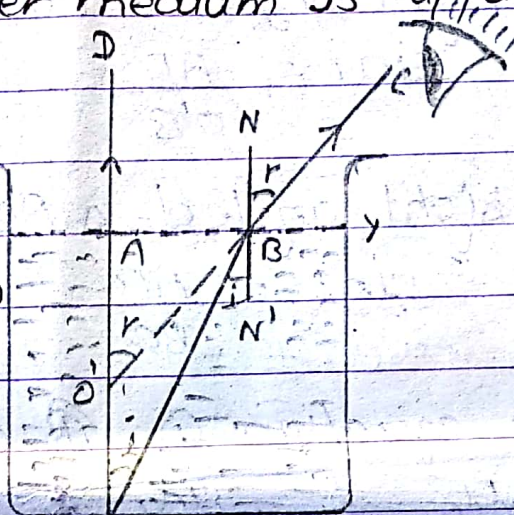


Fig: Real and Apparent depth.

Let us consider an object 'O' is placed at the bottom of beaker containing water (denser medium). Let a ray OA incident on XY normally and goes without bending along AD. Another ray OB incident on XY, and refracted along BC.

Let us produce BO' where O' is the virtual image of object 'O'.

Here,

AO is real depth and AO' is apparent depth.

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The refractive index of water with respect to air is

$$\mu_w = \frac{\sin r}{\sin i} \quad \text{--- (I)}$$

In right angled $\triangle AO'B$,

$$\sin r = \frac{AB}{O'B} \quad \text{--- (II)}$$

In right angled $\triangle AOB$

$$\sin i = \frac{AB}{BO} \quad \text{--- (III)}$$

From equation (I), (II) and (III),

$$\mu_w = \frac{\sin r}{\sin i}$$

$$= \frac{\frac{AB}{O'B}}{\frac{AB}{OB}}$$

$$\mu_w = \frac{OB}{O'B} \quad \text{--- (IV)}$$

Since, angle i and r are very small due to the small aperture of eye lens.

$$OB \approx OA \text{ and } O'B \approx O'A$$

Then

$$\mu_w = \frac{OA}{O'A}$$

$$\therefore \mu_w = \frac{\text{Real depth}}{\text{Apparent depth}}$$

the refractive index of air with respect to water.

$$\mu_{wa} = \frac{\sin i}{\sin r}$$

the refractive index of water with respect to air,

$$\mu_w = \frac{\sin i}{\sin r} \quad \text{--- (I)}$$

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Hence, refractive index of denser medium is defined as the ratio of real depth to the apparent depth.

Apparent displacement (Apparent shift):

The difference between real depth and apparent depth is known as apparent displacement.

It is denoted by 'd'.

$$\therefore \text{Apparent displacement (d)} = \text{Real depth} - \text{Apparent depth}$$

$$d = \text{Real depth} - \frac{\text{Real depth}}{\mu} \quad [\because \mu = \frac{\text{Real depth}}{\text{Apparent depth}}]$$

$$d = \text{Real depth} \left(1 - \frac{1}{\mu}\right)$$

$$\boxed{d = t \left(1 - \frac{1}{\mu}\right)} \text{ where } t = \text{Real depth.}$$

This is the required expression for apparent shift.

$$\mu = \frac{1}{\frac{1}{\mu}} = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin i}{\sin r}$$

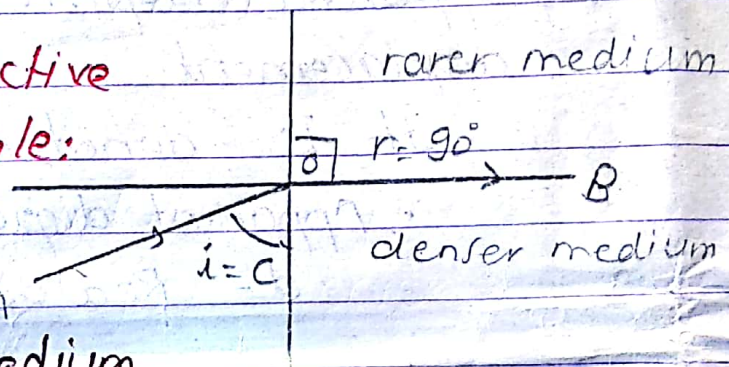
Critical angle:

Critical angle is the angle of incidence in denser medium for which angle of refraction in rarer medium is 90° .

It is denoted by ' c '. Its value depends upon the nature of material.

Relation between refractive index and critical angle:

When the light travels from denser medium to rarer medium,



it bends away from the normal. If we increase

the angle of incidence in denser medium, then angle of refraction in rarer medium also increases. At a particular angle of incidence, the angle of refraction in rarer medium becomes 90° . Then this particular angle of incidence is known as critical angle.

It is denoted by ' c '.

From Snell's law,

$$\text{refractive index } (\mu) = \frac{\sin r}{\sin i}$$

for, Critical angle, $i = c$ and $r = 90^\circ$.

$$\text{or, } \mu = \frac{\sin 90}{\sin c}$$

$$\therefore \boxed{\mu = \frac{1}{\sin c}}$$

This is the relation between refractive index and critical angle.

Note:

For water, $\mu = 1.33$

$$\text{Now, } \mu = \frac{1}{\sin c}$$

$$\text{or, } \sin c = \frac{1}{\mu}$$

$$\text{or, } c = \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$\therefore c = 49^\circ$$

For glass,

$$c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{1}{1.51}\right) (\mu = 1.51)$$

$$\therefore c = 42^\circ$$

For diamond, $\mu = 2.4$

$$c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{1}{2.4}\right) [\because \mu = 2.4]$$

$$c = \cancel{42^\circ} 24.6^\circ$$

Total internal reflection (TIR)

The phenomenon of reflection of light in denser medium when the angle of incidence is greater than critical angle is known as total internal reflection.

It is called total internal reflection because no light is absorbed during this process.

Condition for total internal reflection:

- Light should pass from denser medium to rarer medium.
- Angle of incidence should be greater than critical angle.

Some examples of total internal reflection:

Q) Brilliance of diamond: We know that the refractive index of diamond is 2.4 and hence critical angle is very small (24.6°). The surfaces of diamond are cut such that light in it undergoes multiple internal reflection due to total internal reflection. Hence, light coming out from the same faces makes brilliance of diamond.

(ii) Totally reflecting prism:

An isosceles right angled prism which totally reflect the incident rays falling normally through the base by means of total internal reflection in the refracting faces is

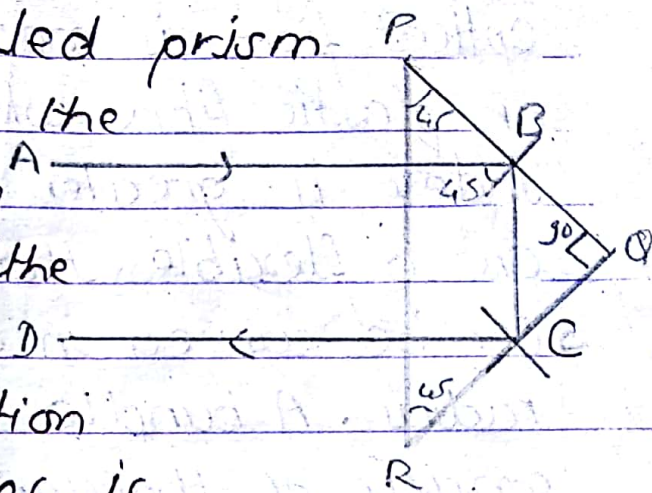


Fig: totally reflecting prism.

known as totally reflecting prism.

When the ray of light AB incident on side PQ at an angle of 45° which is greater than critical angle of glass (i.e. 42°). So total internal reflection takes place and finally light is reflected back along CD as shown in figure. It reverses the incident light by 180° .

It is used in binocular, periscope, reflector etc.

It is more useful than plane mirror because it is ~~to~~ easy to handle and doesnot absorb any ~~short~~ sort of light during total internal reflection.

(iii) Optical fibre:

Optical fibre is made by a bundle of fine glass or plastic fibre whose ~~reflactive~~ reflective index is greater than that of air. They are so flexible that they can be curved in some radius. A bundle consists of thousand of individual fibre each of diameter about 0.02mm to 0.01mm .

When the ray of light is incident on fibre, then the angle of incidence is greater than critical angle so that total internal reflection takes place. Hence, the light can be transmitted from one point to another point. In this process, no light is absorbed. So, it is used in long distance telephone and internet communication.

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Prism:

It is a transparent refracting medium bounded by three plane surfaces. out of three plane surfaces, two are called refracting faces and one is called base. The angle between any two refracting surfaces is called angle of prism. It makes a virtual image of an object.

Refraction of light through glass prism:

Let us consider a ray of light PQ incident at point Q on refracting face AB of glass prism. It is refracted along QR and finally emerges out along RS as shown in figure.

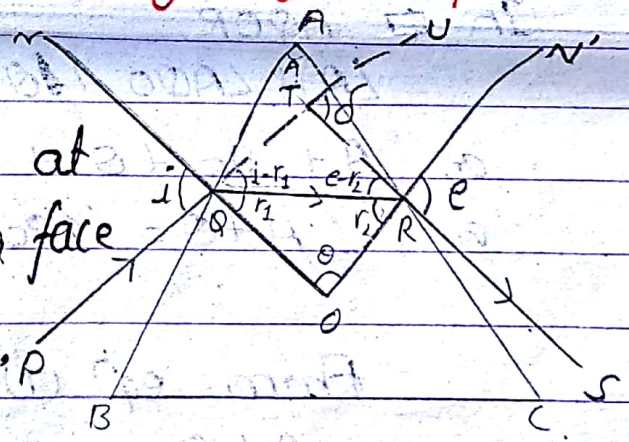


Fig: refraction of light through glass prism

Let, N and N' are normal to face AB and AC respectively.

i = angle of incidence on face AB.

r_1 = angle of refraction on face AB.

r_2 = angle of incidence on face AC

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i_e = angle of refraction emergence on face AC.

The angle made by initial direction of incident ray with produced emergent ray is called angle of deviation and is denoted by δ .

Let, A is angle of prism.

In $\triangle POQ$,

$$r_1 + \theta + r_2 = 180 \text{ (being triangle)} \quad (1)$$

In $\square AQOR$,

$$\angle A + \angle AQO + \theta + \angle ORA = 360^\circ$$

$$\text{or, } A + 90 + \theta + 90 = 360$$

$$\text{or, } A + \theta = 180 \quad (2)$$

From eqⁿ (1) and (2) we get,

$$A + \theta = r_1 + r_2 + \theta$$

$$\text{or, } A = r_1 + r_2 \quad (3)$$

In $\triangle TQR$,

$$\delta = (i - r_1) + (e - r_2) \quad [\because \text{Exterior angle of triangle is equal to sum of interior two opposite interior angle.}]$$

$$\text{or, } \delta = i + e - (r_1 + r_2)$$

$$\therefore \delta = i + e - A \quad (4) \quad (\because \text{from eqⁿ 3})$$

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The angle of deviation depend upon the angle of incidence.

Experimentally, it is found that the angle of deviation goes on decreasing as the angle of incidence increases. At a particular angle of incidence, the angle of deviation become minimum called angle of minimum deviation. Further increasing angle of incidence, the angle of deviation also increases as shown in graph.

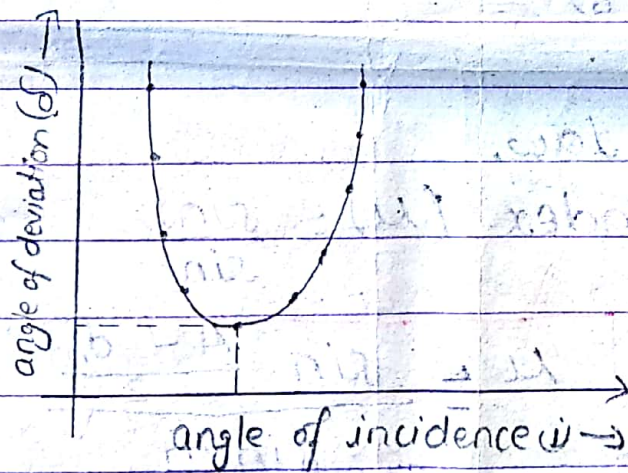


Fig: angle of minimum deviation.

For minimum deviation,

- (i) $i = e$
- (ii) $r_1 = r_2 = r$ (say)
- (iii) $QR \parallel BC$
- (iv) $\delta = \delta_m$

from eqⁿ. 4.

$$d = i + e - A$$

$$d_m = i + i - A$$

$$d_m + A = 2i$$

$$i = \frac{A + d_m}{2} \quad (5)$$

From eqⁿ. 3

$$A = r_1 + r_2$$

$$A = r + r$$

$$A = 2r$$

$$r = \frac{A}{2} \quad (6)$$

from snell's law,

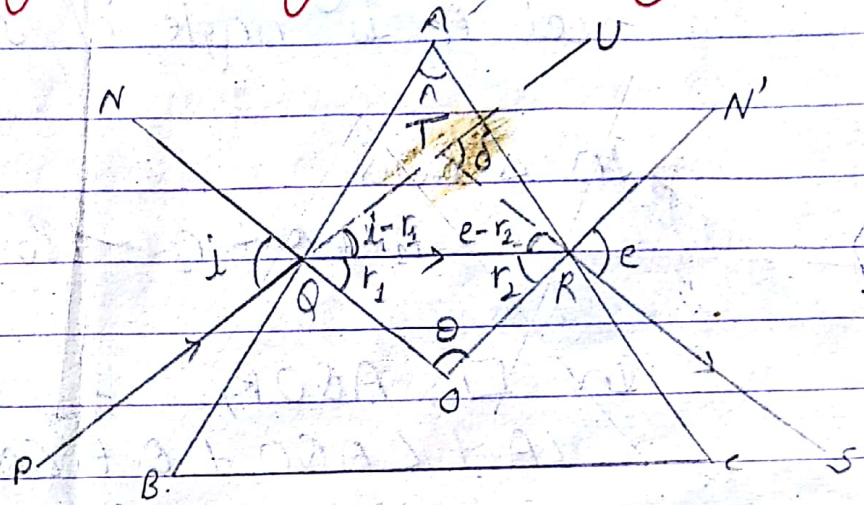
$$\text{Refractive index } (\mu) = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin \frac{A + d_m}{2}}{\sin \frac{A}{2}}$$

This is the relation between refractive index and angle of minimum deviation.

show that the deviation produced by small angled prism is independent of the angle of incidence, provided the angle of incidence is small.

Refraction of light through small angled prism.



Let us consider a ray of light PQ incident at point Q on refracting face AB of ^{small angled} glass prism. It is refracted along QR and finally emerges out along RS as shown in figure.

Let NO and N'O are normal to face AB and AC respectively.

$\angle i$ = angle of incidence on face AB.

$\angle r_1$ = angle of refraction on face AB.

$\angle r_2$ = angle of incidence on face AC.

$\angle e$ = angle of emergence on face AC.

The angle made by initial direction of incident ray with produced emergent ray is called angle of deviation and is

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denoted by δ .

Let A is angle of small angled prism.

In $\triangle ROQ$,

$$r_1 + \theta + r_2 = 180 \text{ (being triangle)} \quad \text{--- (1)}$$

In $\square AQOR$,

$$\angle A + \angle AQO + \theta + \angle ORA = 360^\circ \text{ (being quadrilateral)}$$

$$\text{or, } A + 90 + \theta + 90 = 360^\circ$$

$$\text{or, } A + \theta = 180 \quad \text{--- (2)}$$

From eqⁿ (1) and (2), we get,

$$A + \theta = r_1 + \theta + r_2$$

$$\text{or, } A = r_1 + r_2 \quad \text{--- (3)}$$

In $\triangle TOR$,

$$\delta = (i - r_1) + (e - r_2) \quad \left[\because \begin{array}{l} \text{Exterior angle of} \\ \text{triangle is equal to} \\ \text{the sum of two opposite} \\ \text{interior angle.} \end{array} \right]$$

$$\text{or, } \delta = i + e - (r_1 + r_2)$$

$$\therefore \delta = i + e - A \quad \text{--- (4) } [\because \text{from eqⁿ (3)}]$$

Using Snell's law, on face AB,

$$\mu = \frac{\sin i}{\sin r_1}$$

For small angled prism, $\sin i \approx i$ and $\sin r_1 \approx r_1$,

$$\mu = \frac{i}{r_1}$$

$$i = \mu r_1 \quad \text{--- (5)}$$

Using Snell's law on face AC,

$$\mu = \frac{\sin e}{\sin r_2}$$

For small angled prism, $\sin e \approx e$ and $\sin r_2 \approx r_2$,

$$\mu = \frac{e}{r_2}$$

$$e = \mu r_2 \quad \text{--- (6)}$$

From eqⁿ (4), (5) and (6) we get,

$$\delta = i + e - A$$

$$\delta = \mu r_1 + \mu r_2 - A$$

$$\delta = \mu (r_1 + r_2) - A$$

$$\delta = \mu A - A \quad [\because A = r_1 + r_2]$$

$$\therefore \boxed{\delta = A(\mu - 1)}$$

Hence, deviation produced by small angled prism is independent of angle of incidence.

(b)

- state the factor on which deviation produced by prism depend;

⇒ Deviation produced by prism depend upon the following factors:

- (i) Angle of incidence of light
- (ii) Refracting angle of prism
- (iii) Refractive index of glass of prism.
- (iv) Wavelength of light.

- What is the condition of minimum deviation when a ray of light passes through prism;

⇒ The conditions of minimum deviation when a ray of light passes through prism are given below:

- (i) Angle of incidence must be equal to the angle of emergence i.e. $i = e$.
- (ii) Angle of refraction on 1st face must be equal to the angle of incidence on 2nd face.
- (iii) The refracted ray through the prism must be parallel to the base of prism.

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- Lens:

It is a portion of a transparent medium bounded by two surfaces, at least one surface is curved.

There are two types of lenses.

1. Convex lens
2. Concave lens

- Convex lens.

A lens which is thicker at the centre than at the edges is called convex lens.

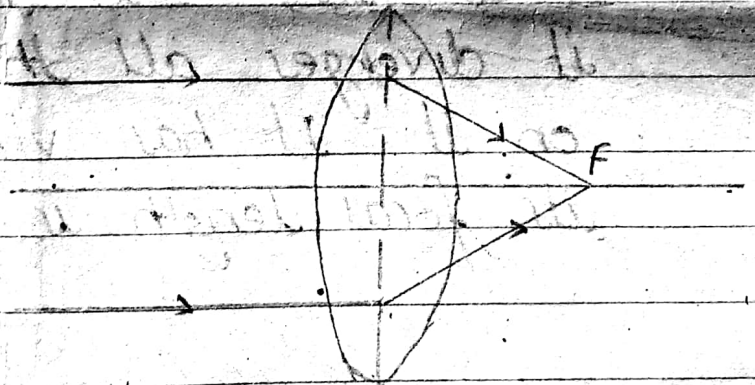


Fig: Convex lens

It is also known as converging lens because it converges all the rays of light incident on it. It has real focus point. So, its focal length is positive.

(2)

- Concave lens:

The lens which is thinner at the centre than at the edges is called concave lens.

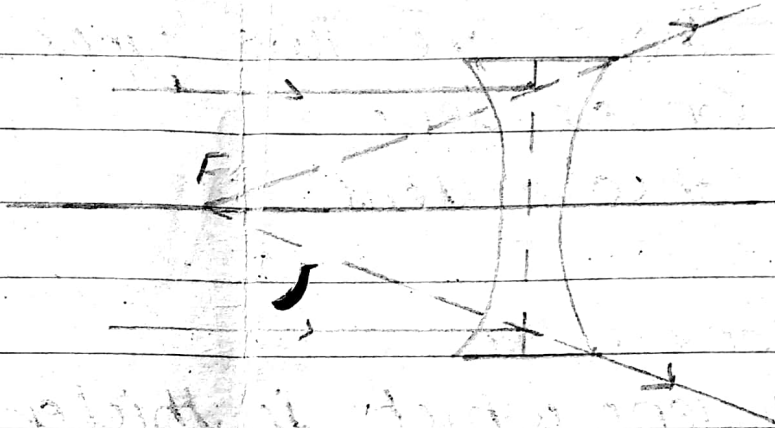


Fig: Concave lens.

It is also known as diverging lens because it diverges all the rays of light incident on it. It has virtual focus point. So, its focal length is negative.

A lens act^{like} a combination of prism. Therefore, when a ray of light passing through the prism, then dev. light is deviated.

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Some terms related to lens.

1. Centre of curvature:

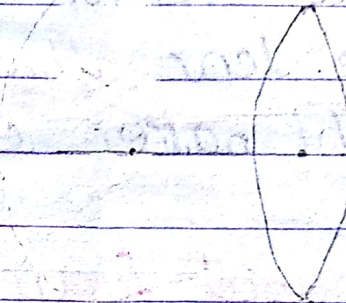


Fig: centre of curvature of convex

The centre of curvature of surface of lens is the centre of sphere of which the surface is a part. It has two centre of curvature.

2. Radius of curvature:

The radius of curvature of surface of lens is the radius of sphere of which the surface is a part. It has two radius of curvature. It is denoted by 'R'.

3. Principal axis:

The line passing through the two centre of curvature of lens is called principal axis.



4. Optical center:

Optical center of a lens is a point in the lens through which a ray of light passes undeviated.

5. Principal focus:

It is a point on principal axis in which rays of light converge or appear to diverge. It is denoted by 'F'.

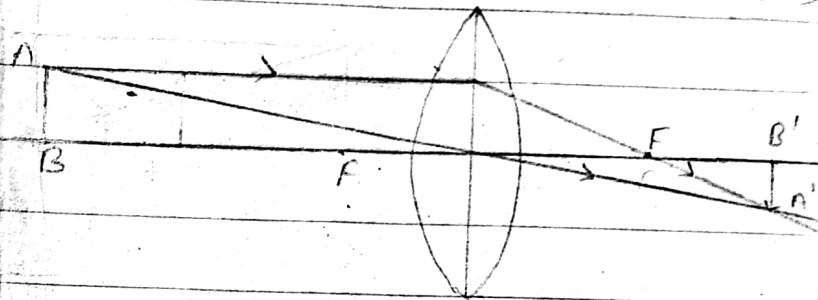
6. Focal length:

The distance between optical center and principal focus^{of} lens is called focal length. It is denoted 'f'. It is positive for convex lens and negative for ~~convex~~ concave lens.

(5)

Rules for obtaining image

1. Convex lens:



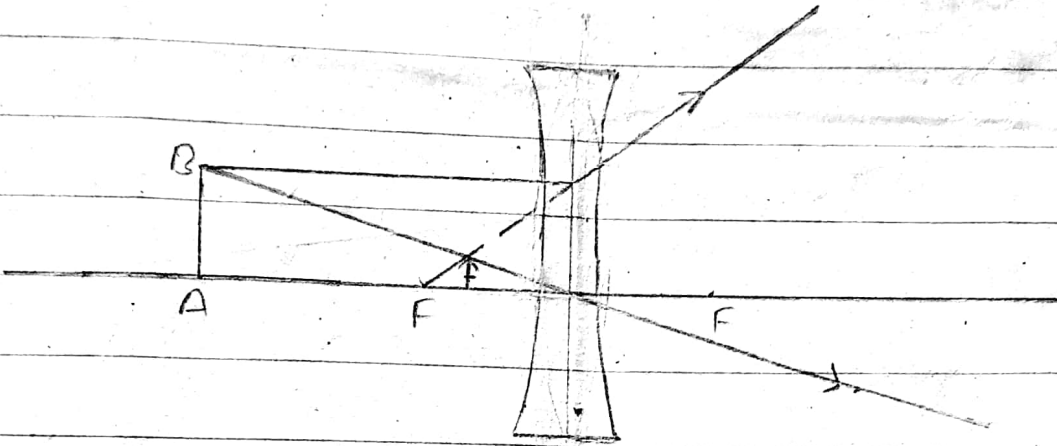
There are following rules for obtaining image:

- (i) An incident ray coming parallel to the principal axis passes through principal focus after refraction.
- (ii) An incident ray passing through focus passes parallel to principal axis after refraction.
- (iii) An incident ray passing through optical centre passes undeviated after refraction.

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2. Concave lens:

There are following rules for obtaining image.



- (i) An incident ray parallel to principal axis appear to diverging from focus after refraction.
- (ii) An incident ray passing through optical centre passes undeviated after refraction.



- Lens formula:

The relation between object distance ' u ', image distance ' v ' and focal length ' f ' of lens is known as lens formula.

1. Lens formula for convex lens:

Let us consider a thin convex lens of focal length ' f ' and optical centre ' C '.

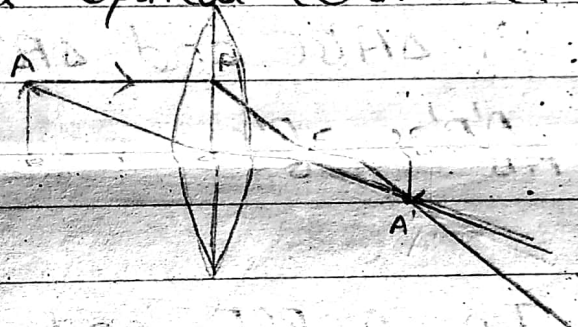


Fig: lens formula for convex lens.

Let, an object AB is placed on principal axis. Let, a ray AP parallel to principal axis passes through focus after refraction and another ray AC passing through optical centre C passes undeviated after refraction.

Image $A'B'$ is formed which is real and inverted as shown in figure.

Let,

'u' = object distance

'v' = image distance

In $\triangle ABC$ and $\triangle A'B'C$

- (i) $\angle ABC = \angle A'B'C$ (being right angle)
- (ii) $\angle ACB = \angle B'CA'$ (being vertically opposite angle)
- (iii) $\angle BAC = \angle B'A'C$ (remaining angle)

$\therefore \triangle ABC$ and $\triangle A'B'C$ are similar triangle

$$\frac{A'B'}{AB} = \frac{B'C}{BC} \quad (1)$$

In $\triangle PCF$ and $\triangle A'B'F$

- (i) $\angle PCF = \angle A'B'F$ (being right angle)
- (ii) $\angle PFC = \angle B'FA'$ (being vertically opposite angle)
- (iii) $\angle CPF = \angle B'A'F$ (remaining angle)

$\therefore \triangle PCF$ and $\triangle A'B'F$ are similar triangle.

$$\frac{A'B'}{PC} = \frac{B'F}{CF}$$

$$\text{or, } \frac{A'B'}{AB} = \frac{B'F}{CF} \quad [\because AB = CP] \quad (2)$$

From eqⁿ (1) and (2), we get

$$\frac{B'C}{BC} = \frac{B'F}{CF}$$

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$$\text{or, } \frac{B'C}{BC} = \frac{B'C - CF}{CF}$$

Here, $B'C = v$, image distance

$BC = u$, object distance

$CF = f$, focal length.

$$\text{or, } \frac{v}{u} = \frac{v-f}{f}$$

$$\text{or, } vf = uv - uf$$

Dividing both sides by uvf , we get

$$\frac{vf}{uvf} = \frac{u}{uvf} - \frac{uf}{uvf}$$

$$\text{or, } \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Magnification:

It is defined as the ratio of size of image to the size of object. It is denoted by m .

(10)

$\therefore$ magnification (m) = $\frac{\text{size of image}}{\text{size of object}}$

$$\text{or } m = \frac{A'B'}{AB}$$

$$\text{or } m = \frac{B'C}{Bc} \quad (\text{from eq 1})$$

$$\therefore m = \frac{v}{u}$$

Note:

If $v = +ve$, then image is real and inverted.
If $v = -ve$, then image is virtual and erect.

2. Lens formula for concave lens.

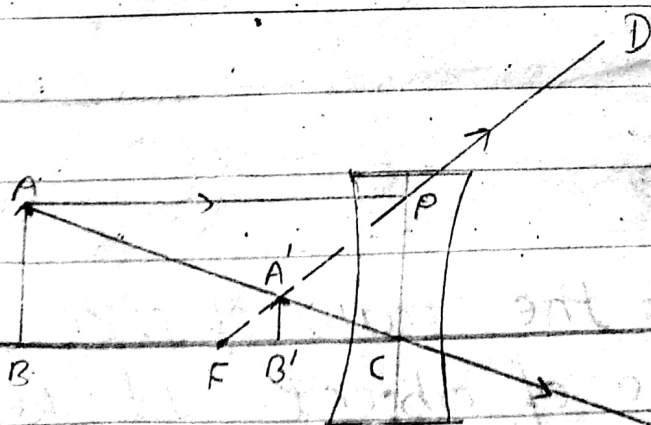
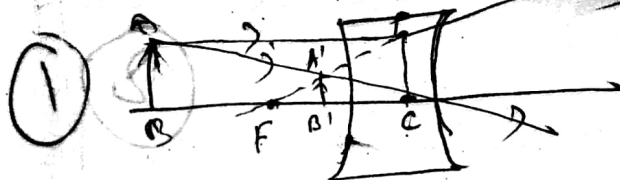


Fig: Concave lens.

Formula for Concave Lens

(11)



Let us consider a concave lens having focal length f and optical centre C . Let an object AB is placed on principal axis.

Let, a ray AP parallel to principal axis refracted along PD and appears to come from focus point F . Another ray AC goes without bending through optical centre. Let image $A'B'$ is formed which is virtual and erect.

In $\triangle ABC$ and $\triangle A'B'C$.

$\angle ACB = \angle A'CB'$ (being common angle)

$\angle ABC = \angle A'B'C$ (being right angle)

$\angle BAC = \angle B'A'C$ (remaining angle)

So, $\triangle ABC$ and $\triangle A'B'C$ are similar triangle.

$$\frac{A'B'}{AB} = \frac{B'C}{BC} \quad \text{--- (i)}$$

In $\triangle PCF$ and $\triangle A'B'F$

$\angle PFC = \angle A'FB'$ (being common angle)

$\angle PCF = \angle A'B'F$ (being right angle)

$\angle FPC = \angle FA'B'$ (remaining angle)

So, $\triangle PCF$ and $\triangle A'B'F$ are similar triangle.

$$\frac{A'B'}{PC} = \frac{FB'}{FC}$$

or $A'B' = FB' \left[\because PC = AB \right] \quad \text{--- (ii)}$

From eq (i) and (ii), we get

$$\frac{B'F}{BC} = \frac{FB'}{FC}$$

$$\text{or, } \frac{B'C}{BC} = \frac{FC - B'C}{FC}$$

$$\text{or, } \frac{-v}{u} = \frac{-f - (-v)}{-f} \quad \left[\begin{array}{l} \because B'C = -v \text{ (for virtual image)} \\ CF = -f \text{ (for concave lens)} \end{array} \right]$$

$$\text{or, } vf = -uf + uv$$

Dividing both sides by uvf , we get

$$\frac{vf}{uvf} = \frac{-uf}{uvf} + \frac{uv}{uvf}$$

$$\text{or, } \frac{1}{u} = -\frac{1}{v} + \frac{1}{f}$$

$$\text{or, } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This is lens formula for concave lens.

Magnification:

It is defined as the ratio of image size to the object size. It is denoted by m .

(13) (3)
Magnification (m) = $\frac{\text{image size}}{\text{object size}}$

or $m = \frac{A'B'}{AB}$

or $m = \frac{B'C}{BC}$ ($\because$ from eqⁿ (i))

$$\therefore m = -\frac{v}{u}$$

This negative sign indicates that the image is virtual and erect.

Power of lens:

It is the ability to converge or diverge the ray of light incident on lens. It is defined as the reciprocal of focal length of lens expressed in meter. It is denoted by P .

$$\text{Power (P)} = \frac{1}{\text{focal length}}$$

$$\therefore P = \frac{1}{f}$$

Its unit is dioptre in SI system. Its value is positive for convex lens and negative for concave lens.

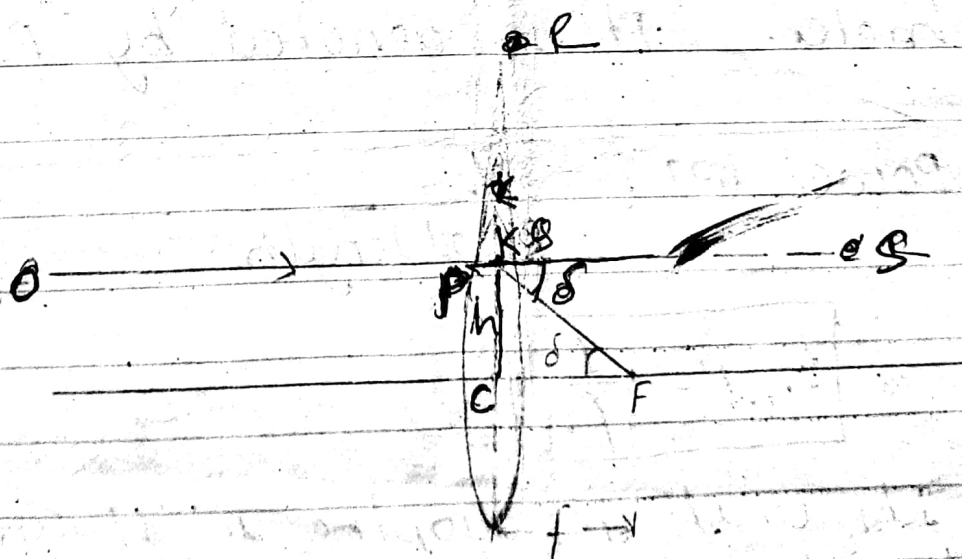
• Lens Maker's formula:

It is the relation between focal length, refractive index of material of lens and radii of curvature of surfaces of lens.

Assumptions:

There are the following assumptions for the derivation of Lens Maker's formula.

- (i) The thickness of lens is very small so that the optical centre is close to the surface.
- (ii) The aperture of lens is small.
- ~~(iii) The incident rays and refracted rays make small angles with the principle axis.~~
- (iv) The deviation produced by the lens is similar to that produced by the small angled prism and the deviation is given by $\delta = A(\mu - 1)$.



(15)

Let us consider a thin lens of focal length f and refractive index μ of the material of lens. Let

Let a ray of light ^{prop} (AB) parallel to the principle axis gets incident at point (B) on as shown in figure. After refraction the ray of light meets the principle axis at point F.

Let, δ = deviation produced by lens.
 $\angle CBF = \angle BFC$ (being alternate angle)

In $\triangle BCF$ $\angle C$

$$\tan \delta = \frac{BC}{CF} = \frac{h}{f}$$

$$\tan \delta = \frac{h}{f}$$

Here, δ is very small

$$\therefore \delta = \frac{h}{f} \quad (i)$$

The deviation produced by lens is equal to the deviation produced.

Let us draw tangents at point (B) and Q such that $\angle BRQ = A$, angle of prism.

The deviation produced by small angled prism is

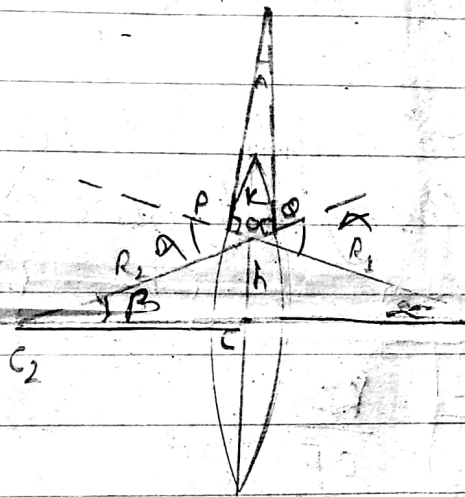
(16)

$$\delta = A(\mu - 1) \quad \text{--- (ii)}$$

From eqⁿ (i) and (ii)

$$\frac{h}{f} = A(\mu - 1)$$

$$\text{or, } \frac{1}{f} = \frac{A}{h}(\mu - 1) \quad \text{--- (iii)}$$



Let, C_1 and C_2 be the centre of curvature and, R_1 and R_2 be the radii of curvature of first surface and second surface respectively. Let us join the point C_1 and P and C_2 and Q such that they meet at @^k point at height h from optical center as shown in figure.

Let α and β be the angle made by PC_1 and QC_2 with principle axis respectively.

Let us draw the tangent at point P and Q.

Since β angle between two tangents forming a prism is equal to β angle between their normals.

from figure,

$$QOQ_1 = A$$

In ΔOC_1C_2

$$A = \alpha + \beta \quad \text{--- (iv)} \quad [\because \text{Exterior angle is equal to the sum of two non-adjacent interior angles.}]$$

In ΔOCC_1 ,

$$\sin \alpha = \frac{h}{OC_1}$$

$$\alpha = \frac{h}{R_1} \quad \text{--- (v)}$$

In ΔOCC_2

$$\sin \beta = \frac{h}{OC_2}$$

$$\beta = \frac{h}{R_2} \quad \text{--- (vi)}$$

Putting the value of α and β in eqⁿ (iv)

$$A = \alpha + \beta$$

$$A = \frac{h}{R_1} + \frac{h}{R_2}$$

$$\frac{A}{h} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{--- (vii)}$$

From eqⁿ (vii) and eqⁿ (iii),

$$\frac{1}{f} = \frac{A}{h} (\mu - 1)$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} \right) (\mu - 1)$$

$$\therefore \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

This is the required expression for Lens Maker's formula.

- A convex lens is emerge in water. Will its focal length change?
- ⇒ The focal length of lens is

$$\frac{1}{f} = (\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{--- (i)}$$

where: R_1 and R_2 are radii of curvature.

μ_g = refractive index of glass.

When it is emerged in water then

$$\frac{1}{f'} = (\omega \mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{--- (ii)}$$

where,

$\omega \mu_g$ = refractive index of glass with respect to water.

Dividing eqⁿ (i) by (ii)

$$\frac{\frac{1}{f}}{\frac{1}{f'}} = \frac{(\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}{(\omega \mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}$$

$$\frac{f'}{f} = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_w} - 1 \right)}$$

Here, $\left(\frac{\mu_g}{\mu_w} - 1 \right) < (\mu_g - 1)$

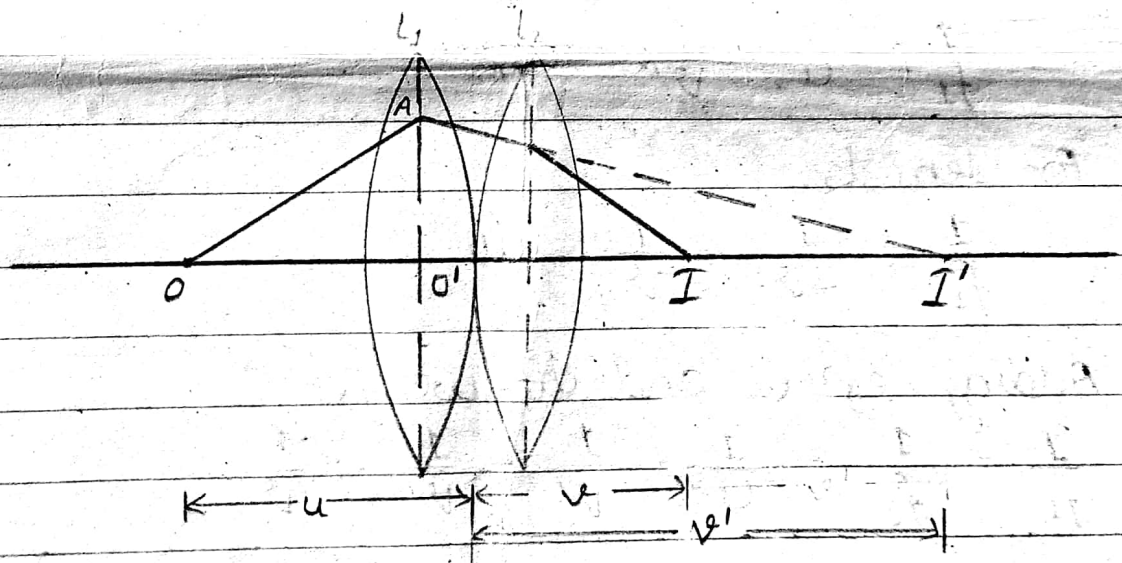
(19)

Here $\therefore f' > f$.

Hence, focal length of lens increases when it is emerge in water.

• Thin lenses in contact:

Derive the combined focal length of two thin lenses in contact.



Let us consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2 placed in contact as shown in figure.

Let a point object O is placed on the principal axis at a distance " u " from lens.

Let a ray OA incident at point A on the

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Lens L_1 . In the absence of lens L_2 , images form at point I' but in the presence of lens L_2 final images form at point I . In this case, the image formed by lens L_1 act as a virtual object for lens L_2 .

Let,

$$OO' = u$$

$$O'I = v$$

$$O'I' = v'$$

For lens L_1 ,

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \text{--- (i)}$$

For lens L_2 ,

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad \text{--- (ii)}$$

Adding eqⁿ (i) and (ii), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v'} - \frac{1}{v'} + \frac{1}{v}$$

$$\text{or, } \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad \text{where, } f \text{ is combined focal length of lens.}$$

(21)

In case of power.

$$P_1 + P_2 = P$$

If 'n' number of lenses are in contact then equivalent combined power is

$$P = P_1 + P_2 + P_3 + \dots + P_n$$

- If a converging lens and a diverging lens having the same focal length be in contact, how will the combination of lenses behave?

Soln Here,

The focal length of combined lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Here,

f_1 = focal length of converging lens.

f_2 = focal length of diverging lens.

then,

$$\frac{1}{f} = \frac{1}{f} + \frac{1}{-f}$$

$$\text{or, } \frac{1}{f} = \frac{1}{f} - \frac{1}{f}$$

$$\text{or, } \frac{1}{f} = 0$$

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or, Power $(P) = 0$.

Hence, combined lens in which one is converging lens and another is diverging lens neither converge nor diverge the ray of light.

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