UNIT 05 | CHAPTER 24 | MODERN PHYSICS

Nuclear Physics

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Nucleus:-

↳ Nucleus was discovevred by Lord Rutherford from α-scattering experiment. An atom consists of positively charge and highly dense central core called nucleus. The diameter of nucleus is order of 10^{-15} m. It consists proton and neutron. proton has +ve charge but neutron is chargeless.

The charge of nucleus is +ze

Where, z = Atomic number (number of proton)

e = Atomic charge

· The radius of the nucleus is directly proportional to the one third power of mass number (A).

i.e, $r \propto A^{1/3}$

$$\Rightarrow \boxed{R = R_0 A^{1/3}}$$

Where, $R_0 = 1.2 \times 10^{-15}$ m.

Nucleon:-

↳ The nucleus contains i.e., proton and neutron are commonly called nucleon.

proton:-

↳ A positively charge particle inside the nucleus whose charge is equal to that of electron is called proton. The mass of proton is 1.67×10^{-27} kg.

Neutron:-

↳ A chargeless particle inside the nucleus is called neutron. The mass of neutron is similarly to that of proton. It has high ~~positively~~ penetrating power due to which slow neutron causes nuclear fission.

Atomic number (z):-

↳ The number of proton inside the nucleus is called atomic number. It is represented by z.

Mass number (A) [Atomic mass]:-

↳ The sum of number of proton and neutron in an atom is called mass number. It is denoted by A.

i.e., A = no. of proton (z) + no. of neutron (N)

$$\therefore A = z + N$$

$$\boxed{N = A - Z}$$

Note:-

The symbol of an atom in nuclear physics represented as

$$_{z}X^{A}$$

Where, z = Atomic number, A = Atomic mass, X = atom

Imp # Properties of Nucleus:-

(i) Nuclear Size:-

↳ The formula for nuclear size is given by;

$$r = r_0 A^{1/3}$$

Where, $r_0 = 1.2 \times 10^{-15}$ m.

Example:-

Nuclear size of Uranium (U^{238})

Here, A = 238

$$\therefore r = 1.2 \times 10^{-15} (238)^{1/3}$$

$$= 1.2 \times (119)^{1/3} \cdot 10^{-15} \times 2^{1/3}$$

$$= 7.436 \times 10^{-15} \text{ m.}$$

5Q) The radius of a nucleus with mass number 16 is 3 fm. Calculate the radius of nucleus with mass number 128.

★ Solution:-

Given, $A_1 = 16$, $r_1 = 3 \text{ fm} = 3\times10^{-15}$ m., $A_2 = 128$, $r_2 = ?$

We have,

$$r = r_0 A^{1/3}$$

So,

$$r_1 = r_0 A_1^{1/3} \text{ ——— (i)}$$

$$r_2 = r_0 A_2^{1/3} \text{ ——— (ii)}$$

Now, Dividing eq^n (ii) by (i), we get;

$$\frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{1/3} \times \frac{r_0}{r_0} = \left(\frac{A_2}{A_1}\right)^{1/3} \times \frac{1.2\times10^{-15}}{1.2\times10^{-15}}$$

or, $$\frac{r_2}{3\times10^{-15}} = \left(\frac{128}{16}\right)^{1/3} = 8^{1/3} = (2^3)^{1/3} = 2$$

$$\Rightarrow r_2 = 6\times10^{-15} \text{ m.} = 6 \text{ fm.}$$

(ii) Nuclear Charge (q):-

↳ A nucleus contains protons and neutrons. protons are positively charge but neutrons are chargele. So, a nucleus is positively charged.
For a nucleus of atomic number 'Z', the nuclear charge 'q' is given by;

$$q = +Ze$$

Where, $e = 1.6\times10^{-19}$ C, is charge of proton.

(iii) Nuclear mass (m):-

↳ The mass of nucleus is equal to sum of masses of neutrons & masses of protons.
Let, m_p be the masses of proton and m_n be the masses of neutrons. then,

Nuclear mass,

$$M = Z m_p + (A-Z) m_n$$

Since, $m_p \approx m_n$

$$\therefore \boxed{M = A m_p}$$

Nuclear density:- (ρ):-

↳ The nuclear mass per unit nuclear volume is called nuclear density.

i.e., Nuclear density (ρ) = $\frac{\text{nuclear mass}}{\text{nuclear volume}}$

Since, nuclear mass, $M = A m_p$

$$\therefore \rho = \frac{A m_p}{\frac{4}{3}\pi r^3}$$

$[\because r = r_0 A^{1/3}]$

$$\Rightarrow \rho = \frac{3 A m_p}{4\pi (r_0 A^{1/3})^3}$$

$$= \frac{3 A m_p}{4\pi r_0^3 A}$$

$$= \frac{3 m_p}{4\pi r_0^3}$$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$

$$\therefore \boxed{\rho = 2.3 \times 10^{17} \ Kg/m^3}$$

This shows that the density of nucleus is very high and independent to it's mass number (A).

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Types of Nuclei:-

(i) **Isotopes:-**

↳ Isotopes are the nuclei having **same atomic number** (z) but **different mass number (A)**. Isotopes of an element have identical chemical behaviour but different physical properties.

Eg:-

(i) Hydrogen:- $_1H^1$, $_1H^2$, $_1H^3$

(ii) Helium:- $_2He^3$, $_2He^4$

(ii) **Isobars:-**

↳ Isobars are the nuclei having **same mass number** but **different atomic number**.

E.g:-

(1) $_1H^3$ & $_2He^3$

(2) $_7N^{14}$ & $_6C^{14}$

(iii) **Isotones:-**

↳ Isotones are the nuclei having the same neutron numbers.

(1) $_2He^4$ & $_1H^3$ (N=2)

(2) $_1H^2$ & $_2He^3$ (N=1)

Mass Defect:-

↳ The neutrons and protons when combine to form the nucleus, it's mass decreases. The decrease in mass is called mass defect.

Let, 'z' be the atomic number and 'A' be the mass number (atomic mass) of an element. Again Let m_p, m_n and M_n be the masses of proton, neutron, and nucleus respectively.

$\therefore$ Mass defect (Δm) = Sum of masses of proton & neutron − mass of nucleus

$$= [Zm_p + (A-Z)m_n] - M_n$$

Packing Fraction [Specific mass defect]:-

↳ The mass defect per nucleon is called packing fraction.

i.e., packing fraction, $P = \frac{\Delta m}{A}$

Q) Calculate the mass defect & packing fraction of $_2He^4$. [(mass of proton = 1.0073 a.m.u.), mass of neutron = 1.0087 a.m.u. and mass of He-nucleus = 4.0015 a.m.u.]

★ Solution:-

Given; Atomic number (Z) = 2, Atomic mass (A) = 4
mass of proton (m_p) = 1.0073 a.m.u., mass of neutron (m_n) = 1.0087 a.m.u.
mass of nucleus (M_n) = 4.0015 a.m.u., mass defect (Δm) & packing fraction (P) = ?

We have, $\Delta m = [Zm_p + (A-Z)m_n] - M_n$

$$= [2 \times 1.0073 + (4-2)1.0087] - 4.0015$$

$$= (2.0146 + 2.0174) - 4.0014$$

$$= 4.032 - 4.0015$$

$$\Rightarrow \Delta m = 0.0305 \text{ a.m.u.}$$

Again;

$$\Rightarrow P = \frac{\Delta m}{A} = \frac{0.0305}{4} = 0.007625 \text{ a.m.u.}$$

Hence, mass defect (Δm) = 0.0305 a.m.u. #
& packing fraction (P) = 0.007625 a.m.u.

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Einstein mass-energy relation:-

↳ Einstein established mass energy relation by his theory of relativity. When a body of mass is moving with speed then energy equivalent to mass is given by:

$$E = mc^2$$

Where, m = mass & c = speed of light in vaccum.

Energy corresponding to mass defect is,

$$E = \Delta m c^2$$

Where,

$$\Delta m = [Z m_p + (A-Z) m_n] - m_n$$

Significance of Einstein mass energy relation:-

(i) It gives a universal equivalance between mass and energy. It means that mass is appear as an energy or vice-versa.

(ii) By Einstein mass energy relation the energy equivalent to 1 a.m.u. is 931 MeV.

Atomic mass unit (a.m.u.):-

↳ The masses of atoms, nucleus and fundamental particles (electrons, protons, neutrons etc) are too small, so expressed in a unit called atomic mass unit.

Imp # 1 a.m.u.:-

↳ It is defined as $\frac{1}{12}$ of the mass of one carbon (C^{12}) atom.

We know;

12 gm of Carbon (C^{12}) Contains 6.023×10^{23} atoms

So,

6.023×10^{23} atoms = 12 gm

$$1 \text{ atom of carbon} = \frac{12}{6.023 \times 10^{23}} \text{ gm}$$

$\therefore$ 1 a.m.u. $= \frac{1}{12}$ of mass of 1 atom of carbon

$$= \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}}$$

$$= 1.67 \times 10^{-24} \text{ gm}$$

$$= 1.67 \times 10^{-27} \text{ kg}$$

But, by Einstein's mass energy relation,

$$E = mc^2$$

Where, $c = 3 \times 10^8$ (velocity of light)

So,

Energy with 1 amu,

$$= 1.67 \times 10^{-27} (3 \times 10^8)^2$$

$$= 1.494 \times 10^{-10} \text{ J}$$

$$= \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV} \quad [1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$= 931 \times 10^6 \text{ eV}$$

$$= 931 \text{ MeV} \quad [1 \text{ MeV} = 10^6 \text{ eV}]$$

Thus, the energy equivalent to 1 a.m.u. is 931 MeV. #

Binding Energy:-

↳ The energy equivalent to the mass defect is known as the binding energy of the nucleons. In other word, the energy equivalent to the mass defect when nucleons bind together to form an atomic nucleus.

When a nucleus is formed, the mass of the nucleons forming it decreases as mass defects. Thus, the binding energy of a nucleus

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is the minimum energy required to break up the nucleus into separate nucleons.
If 'Δm' (in kg) be the mass defect then according to Einstein's mass energy relation, the binding energy 'BE' is given by:

$$B.E. = (\Delta m)\, c^2$$
$$= [\{Zm_p + (A-Z)m_n\} - M_n]\, c^2 \quad \text{(in Joule)} \text{---} (i)$$

If Δm is in a.m.u. then,

$$B.E. = \Delta m \times 931 \text{ MeV} \text{---} (ii)$$

Q.) Calculate the B.E. of $_2He^4$. [Mass of proton (m_p) = 1.007277 a.m.u., mass of neutron (m_n) = 1.008666 amu & mass of $_2He^4$ (M_n) = 4.001509 a.m.u.]

★ Solution,
Given, $Z = 2$, $A = 4$, $m_p = 1.007277$ amu, $m_n = 1.008666$ amu & $M_n = 4.001509$ amu. , B.E. = ?

We know;

$$B.E. = \Delta m \times 931 \text{ MeV} = [\{Zm_p + (A-Z)m_n\} - M_n] \times 931 \text{ MeV}$$
$$= [\{2 \times 1.007277 + (4-2)1.008666\} - 4.001509] \times 931$$
$$= 0.030377 \times 931 \text{ MeV.}$$
$$\text{i.e., } B.E. = 28.28 \text{ MeV}$$

Thus, B.E. of $_2He^4$ = 28.28 MeV. #

Q.) Calculate the B.E. of a nitrogen nucleus in MeV from the following data.

★ $m_p = 1.00783$ amu, $m_n = 1.00867$ amu, $M_n(_7N^{14}) = 14.00307$ a.m.u.
Now, $(Z = 7, A = 14)$

$$B.E. = \Delta m \times 931 \text{ MeV} = [\{Zm_p + (A-Z)m_n\} - M_n] \times 931 \text{ MeV}$$
$$= [\{7 \times 1.00783 + (14-7) \times 1.00867\} - 14.00307] \times 931 \text{ MeV}$$
$$\therefore B.E. = 104.67 \text{ MeV. } \& \ \Delta m = 0.11243 \text{ amu. } \#$$

Binding Energy per nucleon:-

↳ The ratio of binding energy of nucleus and A's mass number is called binding energy per nucleon.

i.e, $$\text{B.E. per nucleon} = \frac{\text{Binding Energy}}{\text{mass number}}$$

$$\therefore \text{B.E. per nucleon} = \frac{BE}{A}$$

The variation of binding energy per nucleon with mass number is as shown in Figure;

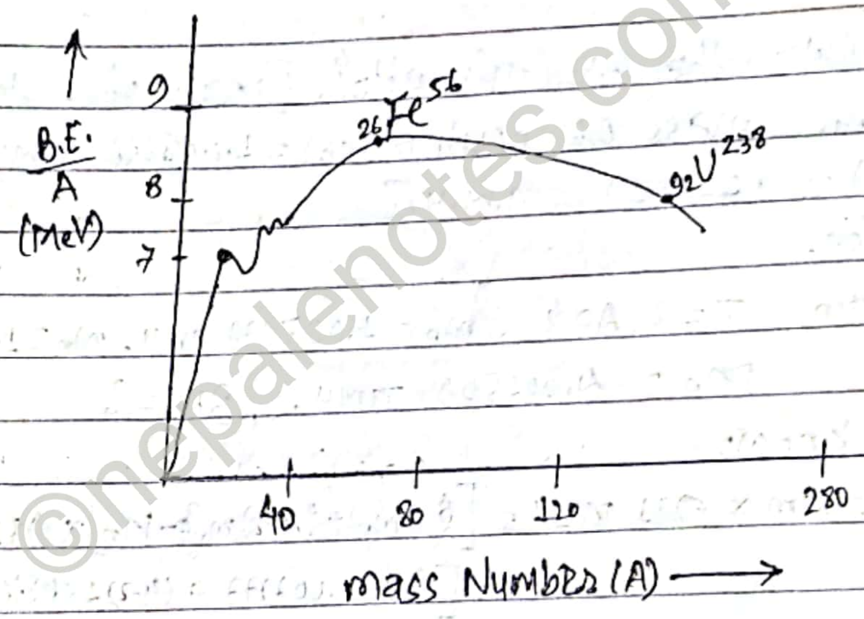

Fig:- Variation of B.E. per nucleon with mass number

The B.E. per nucleon curve is linear for lighter element. The graph rise linearly at first and attains maximum 8.79 MeV for iron and then it drops slowly to 7.6 MeV for Uranium.

Greater the value of B.E. per nucleon more stable the nucleus. So, according to B.E. curve, intermediate elements are more stabe than higher and heaver nuclei. B.E. per nucleon is least for Hydrogen and maximum for iron (Fe^{56}). Thus, Fe^{56} is more stable than hydrogen.

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Nuclear reaction:-

↳ Any reaction involving a change in an atomic nucleus is called nuclear reaction.

Example:- If Nitrogen ($_7N^{14}$) is bombarded by particle ($_2He^4$) then the nuclear reaction is as:

$$_7H^{14} + {_2He^4} \longrightarrow {_8O^{17}} + {_1H^1} + \text{energy}$$

Q-Value of Nuclear Reaction:-

↳ The energy change occuring in a nuclear is deem as the Q-value of nuclear reaction. So the energy released or absorbed during a nuclear reaction is called nuclear reaction energy ~~reaction~~ or energy balance or Q-value of nuclear reaction.

$$A + B \longrightarrow a + b + Q$$

$$\boxed{\text{Q-Value} = (\text{mass of reactant} - \text{mass of product})\, c^2}$$

This formula is used when mass is given in kg.

If mass is in a.m.u. Then Q-Value is given by:

$$\boxed{\text{Q-Value} = (\text{mass of reactant} - \text{mass of product}) \times 931 \text{ meV}}$$

Note:-

⇒ If Q-Value is positive then energy is released and the reaction is called exothermic reaction.

⇒ If Q-Value is negative then energy is absorbed and the reaction is called endothermic reaction.

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Q.) Find the Q-Value and nature of reaction of given nuclear reaction:-

① $\boxed{{}_3Li^7 + {}_1H^1 = 2({}_2He^4) + Q}$

★ Solution.

mass of ${}_3Li^7$ = 7.01822 amu, mass of ${}_2He^4$ = 4.00387 amu

& mass of ${}_1H^1$ = 1.00814 amu. & Q-Value = ?

Now;

Q-Value = (mass of reactant – mass of product) × 931 meV

= [(7.01822 + 1.00814) – 2(4.00387)] × 931 meV

= (8.02636 – 8.00774) × 931 meV

= 0.01862 × 931 meV

⇒ Q-Value = 17.33522 meV #

Here, Q-Value is positive. So, energy is released and reaction is called exothermic reaction. #

② $\boxed{{}_7N^{14} + {}_2He^4 = {}_8O^{17} + {}_1H^1 + Q}$

★ Solution:-

mass of ${}_7N^{14}$ = 14.00753 amu. & Mass of ${}_8O^{17}$ = 17.0450 amu.

mass of ${}_2He^4$ = 4.00387 amu. Mass of ${}_1H^1$ = 1.00814 amu.

Now;

Mass of reactant = (14.00753 + 4.00387) amu = 18.0114 a.m.u. &

Mass of product = (17.0450 + 1.00814) amu = 18.05324 amu.

Then,

Q-Value = [mass of reactant – mass of product] × 931 meV.

= (18.0114 – 18.05324) × 931 meV

⇒ Q-Value = –38.95304 meV #

Here, Q-Value is negative, so, energy is absorbed and reaction is endothermic type. #

Nuclear Fission:- (e.g.:- Atom bomb)

→ The process of spliting of a heavy nucleus into lighter nucleus with the released of larger amount of energy is known as nuclear fission.

For example:- If $_{92}U^{235}$ is bombared with slow neutron an unstable compound nucleus is formed which further break into barium & krypton.

$$_0n^1 + {}_{92}U^{235} \longrightarrow {}_{92}U^{236} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + \text{Energy}(Q)$$

* **For the calculation of energy released in nuclear fission of Uranium:-**

We have,

mass of neutron ($_0n^1$) = 1.008665 a.m.u.

mass of Uranium ($_{92}U^{235}$) = 235.04393 a.m.u.

mass of Barium ($_{56}Ba^{141}$) = 140.9139 a.m.u.

mass of Krypton ($_{36}Kr^{92}$) = 91.8973 a.m.u.

Now;

Energy released = (Total mass of reactant − Total mass of product) × 931 m

$= [(1.008665 + 23.04393) - (140.9139 + 91.8973)] \times 931$ meV

$= (236.05298 - 235.8373) \times 931$ meV

$= 200.2$ meV

Again,

Energy released per nucleon in nuclear fission;

$= \dfrac{200.2}{235}$

$= 0.85$ meV.

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* To Calculate Energy released by 1 kg of Uranium:-

↳ We have;

235 gm uranium contains 6.023×10^{23} atoms

1 gm uranium contains $\frac{6.023 \times 10^{23}}{235}$ atoms

1 kg uranium contains $\frac{6.023 \times 10^{23}}{235} \times 10^{3}$ atoms

$= 2.56 \times 10^{24}$ atoms

∴ Energy released by 1 kg of Uranium:

$= 2.56 \times 10^{24} \times 200.2$ MeV

$= 2.56 \times 10^{24} \times 200.2 \times 10^{6} \times 1.6 \times 10^{-19}$ J

$= 820.0192 \times 10^{11}$ J

$= 8.2 \times 10^{13}$ J

Nuclear fusion:- (e.g.:- H_2 bomb)

↳ The process of formation of heavier nuclear by the fusion of two lighter nuclei which released large amount of energy is called nuclear fusion.

For e.g.:- When two Deutron fused together form a Helium nucleus with the released of energy.

i.e., $_1H^2 + {_1H^2} \longrightarrow {_2He^4} + Q$

We have,

mass of $_1H^2$ = 2.014102 a.m.u.

mass of $_2He^4$ = 4.002604 a.m.u.

Now;

Energy released = (Total mass of reactant - Total mass of product) × 931 MeV

$= [(2.014102 + 2.014102) - (4.002604)] \times 931$ MeV

$= (4.028204 - 4.002604) \times 931$ MeV

$= 23.8$ MeV

Hence, Energy released per nucleon of fusion reaction (Hydrogen bomb) is greater than that of nuclear fission. (Atom bomb).

Thus, Hydrogen bomb is more powerful than atom bomb

Numericals:-

Q. ① The most common isotopes of uranium $_{92}U^{238}$, has atomic mass 238.050783 u. Calculate the ⓐ mass defect. ⓑ binding energy. ⓒ Binding energy per nucleon.
[mass of proton = 1.007825 u., mass of neutron = 1.008665 u.]

Solution:-

Atomic no. (z) = 92, Atomic mass (A) = 238, mass of proton (m_p) = 1.007825 u, mass of neutron (m_n) = 1.008665 u., mass of Uranium atom (M_n) = 238.050783 u.

ⓐ Mass defect (Δm) = ?

We have,

$$\Delta m = [Zm_p + (A-Z)m_n] - M_n$$
$$= [92 \times 1.007825 + (238-92) \times 1.008665] - 288.050783$$
$$= 1.9342 \text{ u.}$$

$\therefore$ Mass defect is 1.9342 u. #

ⓑ B.E. = ?

We have, $BE = (\Delta m) \times 931$ MeV
$= 1.9342 \times 931$ MeV
$= 1800.75$ MeV

$\therefore$ BE. = 1800.75 MeV. #

ⓒ $\frac{BE}{A} = ?$

Now, $\frac{BE}{A} = \frac{1800.75}{238} = 7.566$ MeV

$\therefore$ Binding energy per nucleon is 7.566 MeV. #

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Q.② Calculate the Q-value of the reaction and mention the type of reaction (endothermic or exothermic).

$_2He^4 = 4.00377$ amu. & $_8O^{17} = 17.00450$ amu.
$_7N^{14} = 14.00783$ amu. $_1H^1 = 1.00814$ amu.

★ Solution:-
The reaction will be;

$$_7N^{14} + _2He^4 \longrightarrow _8O^{17} + _1H^1 + Q$$

Now;
Q-Value = [Total mass of reactant − Total mass of product] × 931 meV
$= [(14.00783 + 4.00377) - (17.00450 + 1.00814)] \times 931$ meV
$= (18.01160 - 18.01264) \times 931$ meV
$= -0.96824$ meV
$= -0.96824 \times 10^6 \times 1.6 \times 10^{-19}$ J
$= -1.55 \times 10^{-13}$ J

Hence, Q-Value is -1.55×10^{-13} J. Since, Q-value is negative so, the given reaction is endothermic.

Q.③ What will be the amount of energy released in the fusion of three alpha particles into a C nucleus. If mass of He^4 & C^{12} nuclei are respec 4.00263 amu & 12 amu.

★ The reaction will be;

$$_2He^4 + _2He^4 + _2He^4 \longrightarrow _6C^{12} + Q$$

Now;
Q-Value = [Total mass of (reactant − product)] × 931 meV
$= [(3 \times 4.00263) - 12] \times 931$ meV
$= 7.34559$ meV

Hence, the amount of energy released is 7.34559 MeV #
OR, 7.34559 MeV $= 7.34 \times 10^6 \times 1.6 \times 10^{-19}$ J
∴ Energy released $= 11.75 \times 10^{-13}$ J #

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Q.4 The energy liberated in the fission of a single uranium-235 atom is 3.2×10^{-11} J. Calculate the power production corresponding to the fission of 1.5 kg of uranium per day.

Solution:-

Given; Energy liberated by the fission of a single U^{235} $= 3.2 \times 10^{-11}$ J

time (T) = 1 day = $24 \times 60 \times 60$ sec = 86400 sec

Now, we have,

235 gm of uranium = 6.023×10^{23} atoms

1 gm of uranium = $\frac{6.023 \times 10^{23}}{235}$ atoms

1.5 kg of uranium = $\frac{6.023 \times 10^{23}}{235} \times 1500$ atoms

$= 38.44 \times 10^{23}$ atoms

So,

total energy released by fission of 1.5 kg of uranium

$= 38.44 \times 10^{23} \times 3.2 \times 10^{-11}$ J

$= 1.23 \times 10^{14}$ J

Again;

power production = $\frac{\text{Energy released}}{\text{time}}$

$= \frac{1.23 \times 10^{14}}{86400}$

$= 1.42 \times 10^{9}$ Watt

Hence, power production is 1.42×10^{9} Watt.

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Q5 The energy released by fission of one U^{235} atoms is 200 MeV. Calculate the energy released in KWh, when one ~~given~~ gram of Uranium undergoes fission.

A Solution:-

Given;

Energy liberated by the fission of single $U^{235} = 200$ MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-11}$ J

Time (t) = 1 day = 24×60×60 = 86400 sec.

Now;

235 gm of Uranium = 6.023×10^{23} atoms

1 gm of Uranium = $\frac{6.023 \times 10^{23}}{235}$ atoms

So,

$= 2.56 \times 10^{21}$ atoms

total energy released by the fission of 1 gm Uranium;

$= 3.2 \times 10^{-11} \times 2.56 \times 10^{21}$ J

$= 8.19 \times 10^{10}$ J

∴ Energy released $= 8.19 \times 10^{10}$ J [∵ W = J/s]

$= 8.19 \times 10^{10}$ Ws

$= \frac{8.19 \times 10^{10}}{10^3}$ KWs

$= \frac{8.19 \times 10^{10}}{10^3 \times 60 \times 60}$ KWh

$= 2.275 \times 10^4$ KWh //

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Q6) A city requires 10^7 watts of electrical power on the average. If this is to be supplied by a nuclear reactor of efficiency 20%. Using $_{92}U^{235}$ as the fuel source, Calculate the amount of fuel required per day. [Energy released per fission $_{92}U^{235}$ = 200 meV]

Solution:-

Given; Output power (P_{out}) = 10^7 watts, input power (P_{in}) = ?

Efficiency (η) = 20%

mass required = ?, time (t) = 1 day = $24 \times 60 \times 60 = 86400$ sec

Energy released by single atom of $_{92}U^{235}$ = 200 MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-11}$ J

We have,

$$\eta\% = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\text{or, } 20 = \frac{10^7}{P_{in}} \times 100^5$$

$$\Rightarrow P_{in} = 5 \times 10^7 \text{ Watt}$$

$\therefore$ Energy $= P_{in} \times t = 5 \times 10^7 \times 86400 = 4.32 \times 10^{12}$ J

Now;

3.2×10^{-11} J energy = 1 atom

1 J energy $= \frac{1}{3.2 \times 10^{-11}}$ atoms

4.32×10^{12} J Energy $= \frac{1}{3.2 \times 10^{-11}} \times 4.32 \times 10^{12}$ atoms

$= 1.35 \times 10^{23}$ atom.

Again;

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6.023×10^{23} atoms $= 235$ gm

$\therefore 1$ atoms $= \frac{235}{6.023 \times 10^{23}}$ gm

1.35×10^{23} atoms $= \frac{235}{6.023 \times 10^{23}} \times 1.35 \times 10^{23}$ gm

$= 52.67$ gm

$= \frac{52.67}{1000}$ kg

$= 0.05267$ kg

Thus, mass of $_{92}U^{235}$ required is 0.05267 Kg. #

Q(7) A nucleus of $_{92}U^{238}$ disintegrates according to

$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$

Calculate:-

(i) the total energy released in the disintegration process.

Solution:-

~~[mass of $_{92}U^{238}$ = 238.12492 amu, mass of $_{90}Th^{234}$ = 234.11650 amu, mass of $_2He^4$ = 4.00387 a.m.u., 1 a.m.u. = 931 MeV]~~

Here,

[mass of $_{92}U^{238} = 3.859 \times 10^{-25}$ Kg, mass of $_{90}Th^{234} = 3.787 \times 10^{-}$
& mass of $_2He^4 = 6.648 \times 10^{-27}$ Kg]

Now,

Energy released = [Total mass of (reactant − product)] $\times c^2$

$= [3.859 \times 10^{-25} - (3.787 \times 10^{-25} + 6.648 \times 10^{-27})] [3 \times 10^8]$

$= [3.859 \times 10^{-25} - 3.85348 \times 10^{-25}] \times 9 \times 10^{16}$

$= 0.00552 \times 10^{-25} \times 9 \times 10^{6}$ J

$= 4.968 \times 10^{-21}$ J #

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(ii) the K.E. of the α-particle, the nucleus at rest before disintegration.

Solution:-

$$\text{K.E. of } \alpha\text{-particle} = \frac{m_{Th}}{m_{Th} + m_{\alpha}} \times \text{Energy released}$$

$$= \frac{3.787 \times 10^{-25}}{(3.787 \times 10^{-25} + 6.648 \times 10^{-27})} \times 4.968 \times 10^{-21} \text{ J}$$

$$= \frac{18.813816 \times 10^{-46}}{3.85348 \times 10^{-25}} \text{ J}$$

$$= 4.88 \times 10^{-21} \text{ J}$$ #