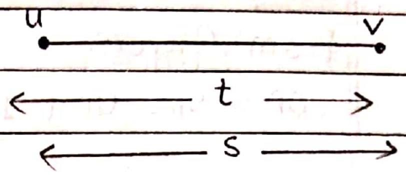
Kinematics

Acceleration (a).

We have,



$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{v-u}{t}$$

$$v - u = at$$

$$v = u + at \quad \text{— (i)}$$

Again $t = \dfrac{v-u}{a}$ — (ii)

$u \Rightarrow$ Initial velocity
$v \Rightarrow$ Final velocity
$t \Rightarrow$ time taken
$s \Rightarrow$ Total displacement covered
$a \Rightarrow$ acceleration

Kinematical Equation.

$$\text{Average velocity} \, (V_{av}) = \frac{\text{Total displacement}}{\text{Time taken}}$$

$$\frac{u+v}{2} = \frac{s}{t}$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s = \frac{(u+u+at)t}{2} = \frac{2ut + at^2}{2}$$

$$\therefore s = ut + \frac{1}{2}at^2$$

Again,

$$\frac{u+v}{2} = \frac{s}{t}$$

Or, $s = \left(\dfrac{u+v}{2}\right)\left(\dfrac{v-u}{a}\right)$

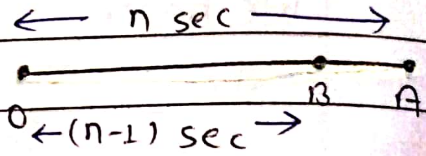
So, $s = \dfrac{v^2 - u^2}{2a}$

$$v^2 - u^2 = 2as$$

$$\therefore v^2 = u^2 + 2as$$

Distance travelled by a body in n^{th} second ($S_{n^{th}}$)

From figure,



$$OA = S_n = un + \frac{1}{2}an^2 \quad \text{—— (i)}$$

$$OB = S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \quad \text{—— (ii)}$$

$$BA = S_{n^{th}} = S_n - S_{n+1} \quad \text{—— (iii)}$$

From (1), (2) and (3),

$$S_{n^{th}} = un + \frac{1}{2}an^2 - \left\{u(n-1) + \frac{1}{2}a(n-1)^2\right\}$$

$$= un + \frac{1}{2}an^2 - \left\{un - u + \frac{1}{2}an^2 - an + \frac{a}{2}\right\}$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + ax - \frac{a}{2}$$

$$= u + an - \frac{a}{2}$$

$$S_{n^{th}} = u + \frac{a}{2}(2n-1)$$

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Equation of motion along

Horizontal	Vertically upward	Vertically downward
$v = u + at$	$v = u - gt$	$v = u + gt$
$S_t = ut + \frac{1}{2}at^2$	$h_t = ut - \frac{1}{2}gt^2$	$h_t = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$	$v^2 = u^2 - 2gh$	$v^2 = u^2 + 2gh$
$S_{nth} = u + \frac{a}{2}(2n-1)$	$h_{nth} = u - \frac{g}{2}(2n-1)$	$h_{nth} = u + \frac{g}{2}(2n-1)$

A man drops a ball from the top of a tower. The ball covers a vertical height of 25m at the last second before it hits the ground. Find the height of the tower.

Solution:

Let the total time taken by the ball to cover total distance be n second. Then,

Initial velocity $(u) = 0\,m/s$

Acceleration due to gravity $(g) = 10\,m/s^2$

Distance travelled in n^{th} second $(S_{nth}) = 25m$

Height of the tower $(h) = ?$

Now,

$$h_{nth} = u + \frac{g}{2}(2n-1)$$

or, $$25 = 0 + \frac{10}{2}(2n-1)$$

or, $$\frac{25}{5} = 2n-1$$

or, $$2n = 6$$

$\therefore$ $n = 3$ seconds

Again,

$$h = ut + \frac{1}{2}gt^2$$

$$= 0 \times 3 + \frac{1}{2} \times 10 \times 3^2$$

$$= 45m$$

Projectile

A body which once thrown into space, moves under the influence of gravity only is called a projectile and such a motion is described as projectile motion and the path followed by a projectile is known as trajectory.

Projectile Fired Horizontally from a Height

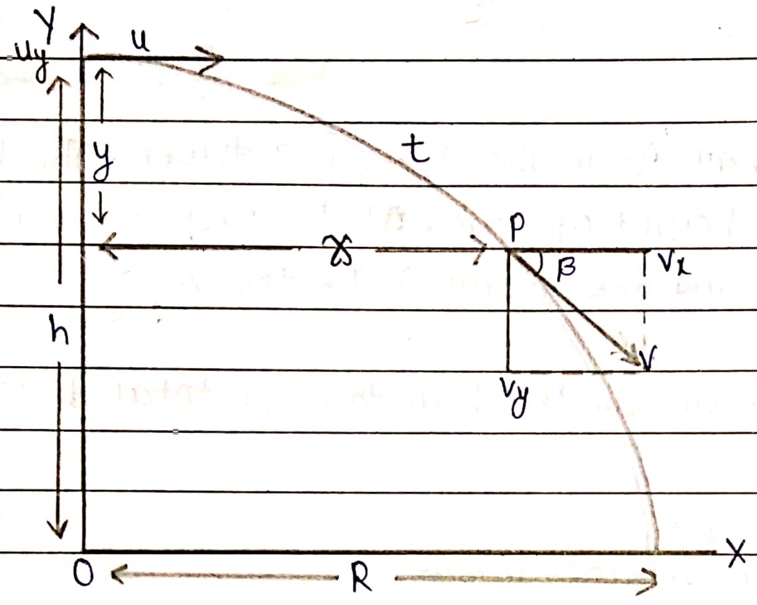

Fig: Projectile fired horizontally

Let us consider a projectile is fired from the top of a tower of height 'h' with a horizontal velocity 'u'. Due to the influence of gravity, it will fall on the ground after some time 'T' covering some distance on the ground following a certain path as shown in figure. Let, the body travels a horizontal distance x and vertical distance y from the initial point in time t sec making its velocity v at this moment at the point p.

↳ Equation of Trajectory

We know,

$$x = ut$$

$$t = \frac{x}{u} \quad \text{--- (i)}$$

Also,

$y = u_y t + \frac{1}{2} g t^2$

$y = \frac{1}{2} g t^2$ --- (ii) $[\because u_y = 0 m/s]$

From equations (i) and (ii),

$y = \frac{1}{2} g \frac{x^2}{u^2}$

$y = \left(\frac{1}{2} \frac{g}{u^2}\right) x^2$ --- (iii)

Equation (iii) represents the equation of parabola. Hence, trajectory of projectile is parabolic.

→ Velocity at any Instant (v)

Let, v be the resultant velocity at point P and β be the angle made by v with v_x. v_x and v_y are velocity components along horizontal and vertical direction respectively.

$v_x = u$

$v_y = u_y + gt$

$v_y = gt$ $[\because u_y = 0]$

$v = \sqrt{v_x^2 + v_y^2}$

Again,

$\tan\beta = \frac{v_y}{v_x}$

$= \frac{gt}{u}$

$\therefore \beta = \tan^{-1}\left(\frac{gt}{u}\right)$

Time of Flight (T)

It is the total time for which the projectile remains in air.
It is denoted by T.

We have,

$$h = u_y T + \frac{1}{2} g T^2$$

$$h = \frac{1}{2} g T^2 \quad [\because u_y = 0]$$

$$T^2 = \frac{2h}{g}$$

$$T = \sqrt{\frac{2h}{g}}$$

Horizontal Range (R)

It is the horizontal distance covered by the projectile during the time of flight.

$$R = uT$$

$$R = u\sqrt{\frac{2h}{g}}$$

Projectile Fired at an Angle with Horizon

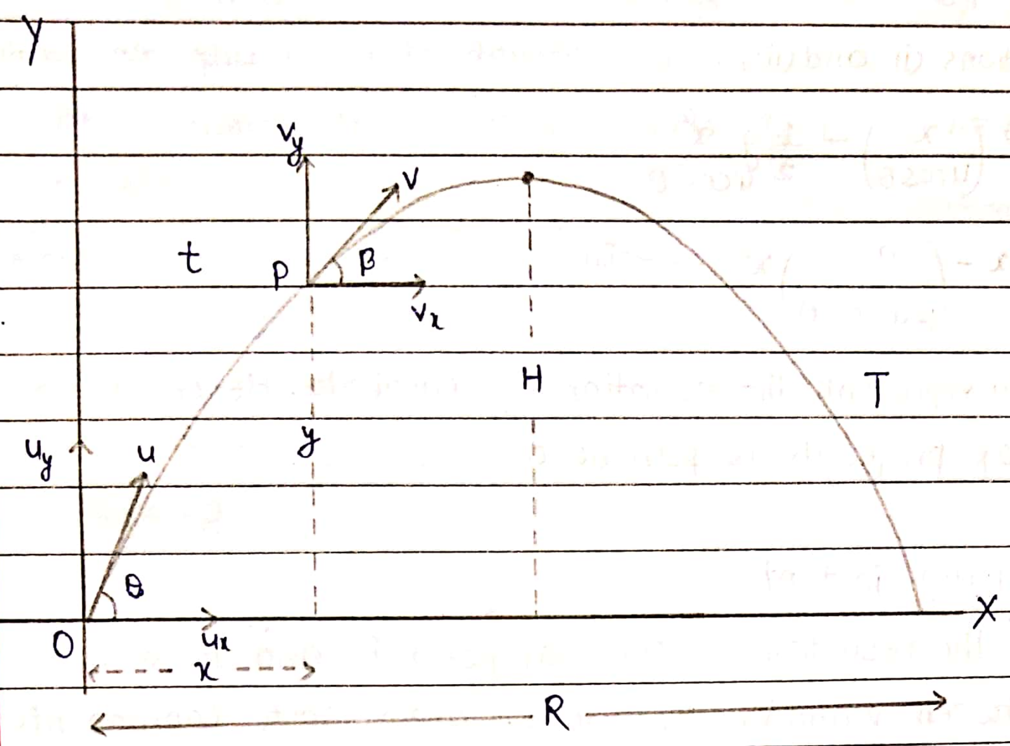

Fig: Projectile fired at certain angle with horizon

Let us consider a body is thrown into space with initial velocity u making an angle θ with horizon as shown in the figure. u_x and u_y are initial velocity components along horizontal and vertical directions respectively. Since, there is no acceleration along horizontal direction, horizontal velocity component remains constant throughout motion. In the projectile motion, the resistance upward by air is neglected. So, $u_y = u\sin\theta$ and $u_x = u\cos\theta$.

Equation of Trajectory

Let P be a point on the path of the projectile such that x and y are horizontal and vertical distance covered in time 't' respectively.

$x = u_x t$

$x = u\cos\theta \cdot t$

$t = \dfrac{x}{u\cos\theta}$ ---- (i)

Now,

$y = u_y t - \frac{1}{2} g t^2$

$$y = (u\sin\theta)\cdot t - \frac{1}{2}gt^2 \text{ --- (ii)}$$

From equations (i) and (ii),

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2 \text{ --- (iii)}$$

Equation (iii) represents the equation of parabola. Hence, the trajectory of projectile is parabolic.

→ velocity at any instant

Let, V be the resultant velocity at point P and β be the angle between V and V_x. V_x and V_y are velocity components along horizontal and vertical directions respectively.

$V_x = u_x = u\cos\theta$

$V_y = u_y - gt$
$= u\sin\theta - gt$

$V = \sqrt{V_x^2 + V_y^2}$

$V = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$ --- (iv)

Again,

$$\tan\beta = \frac{V_y}{V_x}$$

$$= \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\therefore \beta = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right) \text{ --- (v)}$$

Equations (iv) and (v) give the magnitude and direction of the resultant V.

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Maximum height (H)

It is the maximum vertical distance covered by projectile during time of flight. It is denoted by H.

At maximum height vertical velocity component is zero (0).

$v_y' = 0$

Now,

$$v_y^2 = u_y^2 - 2gH$$

$$0 = u^2\sin^2\theta - 2gH$$

$$H = \frac{u^2\sin^2\theta}{2g} \quad \text{--- (vi)}$$

Time of ascend (t')

It is the time taken by projectile to reach maximum height.

$$v_y' = u_y - gt'$$

$$0 = u\sin\theta - gt'$$

$$t' = \frac{u\sin\theta}{g} \quad \text{--- (vii)}$$

Time of Flight (T).

It is the total time for which the projectile remains in air. It is denoted by T.

During time of flight, net vertical height gained by projectile is 0.

$$0 = u_yT - \frac{1}{2}gT^2$$

or $0 = u\sin\theta - \frac{1}{2}gT$ [∵ Dividing both sides by T]

or $T = \frac{2u\sin\theta}{g}$ --- (viii)

↳ Time of descend (t'')

$$t'' = T - t'$$

$$= \frac{2u\sin\theta}{g} - \frac{u\sin\theta}{g}$$

$$= \frac{u\sin\theta}{g} \quad \text{--- (ix)}$$

↳ Horizontal Range (R)

It is the maximum horizontal distance covered by the projectile during the time of flight.

$$R = u_x T$$

$$R = u\cos\theta \cdot \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g} \quad \text{--- (x)}$$

The horizontal range will be maximum if $\sin 2\theta = 1$

i.e. $\sin 2\theta = \sin 90°$

or, $2\theta = 90°$

or, $\theta = 45°$.

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At what angle of projection maximum height and horizontal range are equal?

Solution:

Maximum height (H) = Horizontal Range (R)

or, $\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

or, $\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$

or, $\sin\theta = 4\cos\theta$

or, $\tan\theta = 4$

or, $\theta = \tan^{-1}(4)$

$\therefore \theta = 76^\circ$

$\therefore$ Maximum height and horizontal range are equal at 76° angle of projection.

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NUMERICAL PROBLEMS

1. A swimmer's speed along the river is 20km/hr and upstream is 8 km/h. Calculate the velocity of the stream and the swimmer's possible speed in still water. [2075 GIE Q.No. 9a]

Solution:

Let, the velocity of the swimmer in still water and that of the stream be V_x and V_y respectively.

Speed upstream = $V_x - V_y$

$\therefore V_x - V_y = 8 km/hr$ -- (i)

Speed downstream = $V_x + V_y$

$\therefore V_x + V_y = 20 km/hr$ -- (ii)

From equations (i) and (ii)

$2V_x = 28$

$\therefore V_x = 14 km/hr$

Substituting the value of V_x in equation (ii)

$V_y = 20 - 14$

$\therefore V_y = 6 km/hr.$

$\therefore$ The velocity of the stream is 6km/hr and swimmer's possible speed in still water is 14km/hr.

2. A body is projected upwards making an angle θ with the horizontal with a velocity of 300m/s. Find the value of θ so that the horizontal rangle will be maximum. Hence find its range and time of flight. [2075 Set B Q.No. 9d]

Solution:

Angle of a body projected upward with the horizontal = θ

Initial velocity (u) = 300m/s

We know,

$$\text{Horizontal Range (R)} = \frac{u^2 \sin 2\theta}{g}$$

The horizontal range will be maximum if:

$\sin 2\theta = 1$

$\sin 2\theta = \sin 90°$

$\therefore \theta = 45°$

Now,

$$\text{Horizontal range (R)} = \frac{(300)^2 \times \sin 2(45°)}{10}$$

$$= \frac{90000}{10}$$

$$= 9000\text{m}$$

$$\text{Time of flight (T)} = \frac{2u\sin\theta}{g}$$

$$= \frac{2 \times 300 \times 1/\sqrt{2}}{10}$$

$$= 42.42 \text{ sec.}$$

$\therefore$ The horizontal range will be maximum if θ = 45°. Hence, its range is 9000m and time of flight is 42.42 sec.

3. A stone on the edge of a vertical cliff is kicked so that its initial velocity is 9m/s horizontally. If the cliff is 200m high, calculate: [2074 Supp. Q. No. 9a]

i. time taken by stone to reach the ground.

ii. How far from the cliff the stone will hit the ground?

Solution:

Height of the cliff (h) = 200m

Initial velocity of the stone horizontally (u) = 9m/s

Now,

Time taken by stone to reach ground $(T) = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2 \times 200}{10}}$

$= 6.32$ sec.

Also,

Horizontal Range $(R) = u\sqrt{\frac{2h}{g}}$

$= 9 \times 6.32$

$= 56.92$ m

∴ The time taken by stone to reach ground is 6.32 sec and the stone will hit the ground 56.92 m far from the cliff

4. A projectile is fired from the ground level with a velocity 500 m/s at 30° to horizon. Find the horizontal range, the greatest height and the time to reach the greatest height. [2074 set B / 2073 Suppl / 2066 Q.No. 9a].

Solution:

Initial velocity of a projectile $(u) = 500 m/s$

Angle of the projectile with horizon $(\theta) = 30°$.

Now,

Horizontal range $(R) = \frac{u^2 \sin 2\theta}{g}$

$= \frac{(500)^2 \times \sin 2(30°)}{10}$

$= \frac{250000 \times \sqrt{3}/2}{10}$

$= 21651 m$

Greatest height $(H) = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{(500)^2 \cdot \sin^2 30°}{2 \times 10}$

$= \frac{250000 \times (1/2)^2}{20}$

$= 3125 m$

Time to reach the greatest height $(t) = \frac{u \sin\theta}{g}$

$= \frac{500 \cdot \sin 30°}{10}$

$= \frac{500 \cdot 1/2}{10}$

$= 25$ sec.

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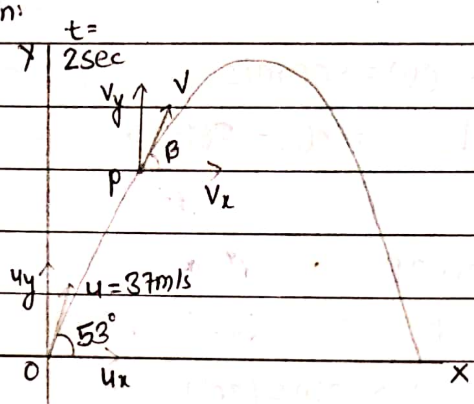
5. A batter hits a baseball so that it leaves the bat with an initial speed 37m/s at an angle of 53°. Find the position of the ball and the magnitude and direction of its velocity after 2 seconds. [2073 set c Q.No. 9a]

Solution:



Let, P be a point on the path of the baseball such that it reaches the point P in 2seconds. v be the resultant velocity at point P. v_x and v_y are velocity components along horizontal and vertical directions respectively. β be the angle made by v with v_x.

$v_x = u_x = 37\cos 53° = 22.27$ m/s

$v_y = u_y - gt = 37\sin 53° - 10\times 2 = 9.55$ m/s

Now,

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(22.27)^2 + (9.55)^2}$$
$$= \sqrt{587.16}$$
$$= 24.23 \text{ m/s}$$

Now,

$$\tan\beta = \frac{v_y}{v_x}$$

$$\tan\beta = \frac{9.55}{22.27}$$

$$\beta = \tan^{-1}(0.43)$$

$$\therefore \beta = 23.27°$$

The magnitude and direction of the velocity of the baseball are 24.23m/s and 23.27° respectively.

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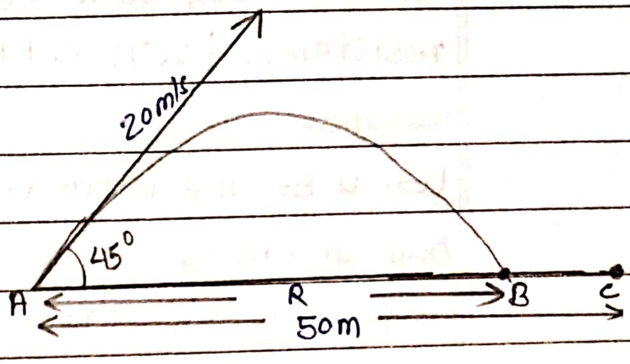
6. A baseball is thrown towards a player with an initial velocity 20 m/s and 45° with the horizontal. At the moment, the ball is thrown to the player is 50m from the thrower. At what speed and in which direction must he run to catch the ball at the same height at which it is released?

Solution:



Initial velocity (u) = 20m/s

Angle of projection (θ) = 45°

Distance between the thrower and the player (AC) 50m

We know,

$$\text{Horizontal Range} = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(20)^2 \times \sin 2(45)^\circ}{10}$$

$$= 40\text{m}$$

And,

$$\text{Time of Flight} = \frac{2u\sin\theta}{g}$$

$$= \frac{2 \times 20 \times \sin 45^\circ}{10}$$

$$= 2.83\text{sec}$$

Now,

The distance required to run = 50m − 40m (BE) 10m

The speed with which the player must run

$$= \frac{\text{Distance covered}}{\text{time taken}} = \frac{10}{2.83} = 3.53\text{m/s}.$$

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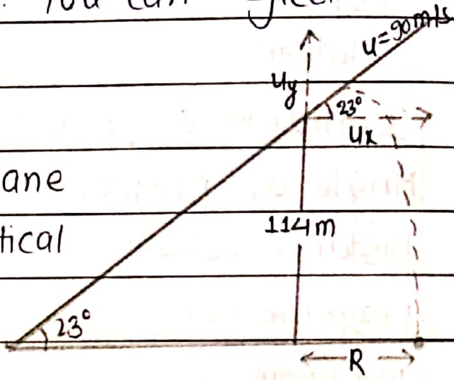
7. An airplane is flying with a velocity of 90m/s at an angle of 23° above the horizontal. When the plane is 114m directly above a dog that is standing on level of ground, a suitcase drops out of luggage compartment, how far from the dog with the suitcase land? You can neglect air resistance. [2072 Set D Q.No. 9a]



Solution.

Let, u be the initial velocity of an airplane and u_x and u_y be its horizontal and vertical components respectively. 'h' be the distance between the flying airplane and a dog on the level of ground and R be the horizontal range.

Then, $u = 90 m/s$

$u_x = 90 \cos 23° = 82.85 m/s$

$u_y = 90 \sin 23° = 35.17 m/s$

$h = 114m$

$R = ?$

Now,

$$-h = u_y t - \frac{1}{2} g t^2$$

or, $$-114 = 35.17t - \frac{1}{\cancel{2}} \times \overset{5}{\cancel{10}} t^2$$

or, $5t^2 - 35.17t - 114 = 0$ which is in the form of $at^2 + bt + c = 0$.

$$\therefore t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{35.17 \pm \sqrt{(-35.17)^2 - 4 \times 5 \times (-114)}}{2 \times 5}$$

$$= \frac{35.17 \pm 59.3}{10}$$

Taking '+ve' sign.

$t = 9.45$ sec.

Now,

$R = u_x t$

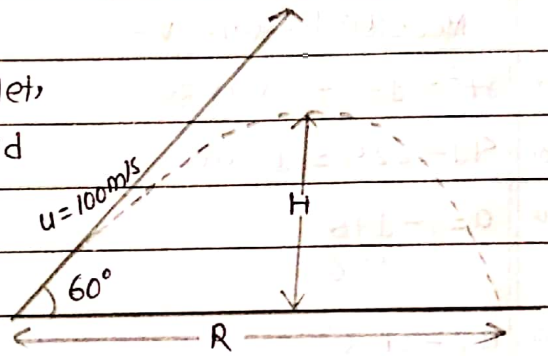
$= 82.85 \times 9.45$

$= 782.71 m.$

$\therefore$ The suitcase will land 782.71 m from the dog.

8. A bullet is fired with a velocity of 100m/s from the ground at an angle of 60° with the horizontal. Calculate the horizontal range covered by the bullet. Also calculate the maximum height attained. [2071 Set C Q.No. 9d]

Solution:



Let, u be the initial velocity of a bullet,

H be the maximum height attained and

R be the horizontal range covered.

Then, $u = 100 m/s$

$H = ?$

$R = ?$

We know,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(100)^2 . \sin 2(60°)}{10}$$

$$= \frac{10000 . \sqrt{3}/2}{10}$$

$$= 866 m$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(100)^2 . \sin^2 60°}{2 \times 10}$$

$$= \frac{10000 . 3/4}{20}$$

$$= 375 m$$

9. A car travelling with a speed of 15m/s is braked and slows down with uniform retardation. It covers a distance of 88m as its velocity reduces to 7m/s. If the car continues to slow down with the same rate, after what further distance will it be brought to rest? [2070 supp. (Set A) Q. No. 9A]

Solution:

Final velocity (v) = 7m/s

Initial velocity (u) = 15m/s

Distance covered (s) = 88m

We know,

$v^2 = u^2 + 2as$

or, $7^2 = 15^2 + 2 \times a \times 88$

or, $49 - 225 = 176a$

or, $a = -\frac{176}{176}$

or, $a = -1 m/s^2$

$\therefore$ Retardation (a) = $1 m/s^2$

Again, If the car continues to slow down with same rate,

$v^2 = u^2 + 2as$

or, $0^2 = 7^2 + 2 \times (-1) \times s$

or, $-49 = -2s$

or, $s = \frac{-49}{-2}$

$\therefore$ $s = 24.5m.$

Hence, the car will be brought to rest after 24.5m.

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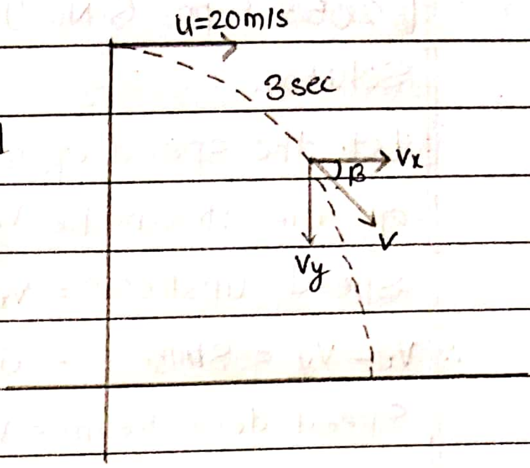
10. A stone is projected horizontally with 20m/s from top of a tall building. Calculate its position and velocity after 3sec neglecting the air resistance. [2070 Supp. (Set B) Q.No. 9a]

Solution:



Let, V be the resultant velocity after 3 sec and V_x and V_y be the horizontal and vertical components respectively. β be the angle between V and V_x.

Initial velocity (u) = 20m/s

Time (t) = 3sec

$V_x = u = 20 m/s$

$V_y = gt = 10 \times 3 = 30 m/s$

We know,

$$V = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{20^2 + 30^2}$$

$$= \sqrt{1300}$$

$$= 36 m/s$$

Also,

$$\tan \beta = \frac{V_y}{V_x}$$

$$\beta = \tan^{-1}\left(\frac{30}{20}\right)$$

$$\beta = 56.3°$$

∴ The velocity of stone after 3sec is 36m/s and its position is 56.3°.

11. A swimmer's speed along the river (down stream) is 20km/h and can swim upstream at 8km/h. Calculate the velocity of the stream and the swimmer's possible speed in still water.
[2059 Supp. Q. No. 9a]

Solution:

Let, the speed of swimmer in still water be v_x and the velocity of the stream be v_y.

Speed upstream = $v_x - v_y$

$\therefore v_x - v_y = 8\ km/hr$ --- (i)

Speed downstream = $v_x + v_y$

$\therefore v_x + v_y = 20\ km/hr$ --- (ii)

From equations (i) and (ii),

$2v_x = 28\ km/hr$

$v_x = 14\ km/hr$

$\therefore$ Speed of swimmer in still water is 14 k

Substituting the value of v_x in equation (ii)

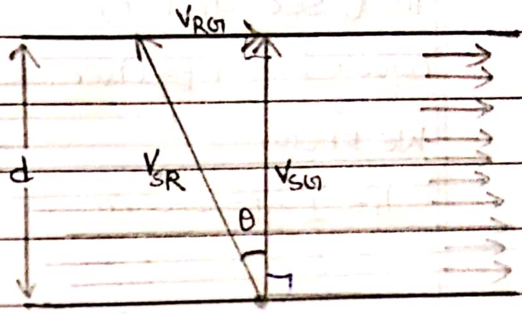
$v_y = (20 - 14)\ km/hr.$

$= 6\ km/hr.$

$\therefore$ The velocity of the stream is 6 km/hr.

12. A man wishes to swin across a river 600m wide. If he can swim at the rate of 4km/hr in still water and the river flows at 2km/hr. Then in what direction must he swim to reach a point exactly opposite to the starting point and when will he reach it? [2069 (set A) Q. No. 9a]

Solution:



Width of river (d) = 600m = 0.6km

velocity of swimmer with respect to river (V_{SR}) = 4km/hr

Velocity of river with respect to ground (V_{RG}) = 2km/hr

Direction that he must swim with the direction of flowing water $(90^\circ+\theta)$ = ?

Time to cross the river (t) = ?

From the figure,

$$\sin\theta = \frac{V_{RG}}{V_{SR}} = \frac{2}{4} = \frac{1}{2}$$

$$\text{or, } \theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$\therefore \theta = 30^\circ$ $\qquad \therefore 90^\circ + \theta = 120^\circ$

Now,

$$V_{SG} = \sqrt{V_{SR}^2 - V_{RG}^2}$$
$$= \sqrt{4^2 - 2^2}$$
$$= \sqrt{12}$$
$$= 3.46 \text{ km/hr}$$

Then,

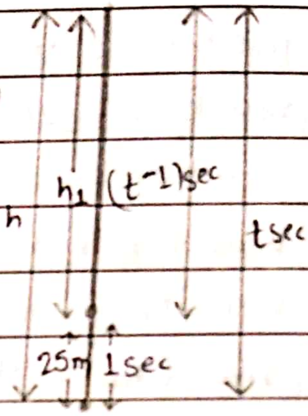
$$t = \frac{d}{V_{SG}} = \frac{0.6}{3.46}$$
$$= 0.17 \text{ hr}$$
$$= 10.4 \text{ min.}$$

$\therefore$ He must swim at 120° and he will reach the point exactly opposite to the starting point after 10.4 minutes.

13. A body falls freely from the top of a tower and during the last second of its fall, it falls through 25m. Find the height of tower. [2068 (set B) Q. No. 9c]

Solution:

Let, h be the total height of the tower covered in t sec and h_1 be the height of the tower covered in (t-1)sec.



We know,

$$h = \frac{1}{2}gt^2 \quad --(i)$$

$$h_1 = \frac{1}{2}g(t-1)^2 \quad --(ii)$$

Subtracting (ii) from (i),

$$h - h_1 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$$

or, $25 = \frac{1}{2} \times 10 \left[t^2 - (t^2 - 2t + 1)\right]$

or, $\frac{25}{5} = t^2 - t^2 + 2t - 1$

or, $5 + 1 = 2t$

$\therefore$ $t = 3$ sec.

Substituting t = 3sec in equation (i)

$h = \frac{1}{2} \times 10 \times 3^2$

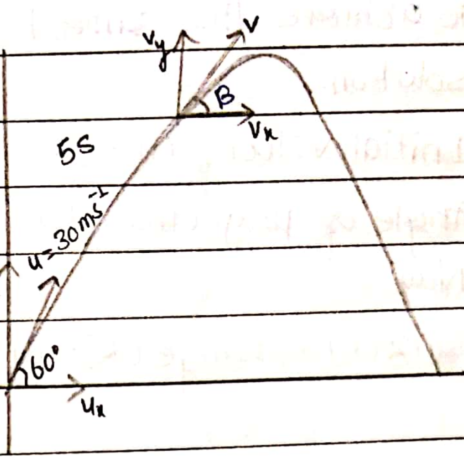
$= 5 \times 9$

$= 45$m.

$\therefore$ The height of the tower is 45m.

14. A projectile is launched with an initial velocity of $30ms^{-1}$ at an angle of $60°$ above the horizontal. Calculate the magnitude and direction of its velocity 5s after launch. [2069 old (set B) Q.No.26]

solution:



Let, u be the initial velocity and u_x and u_y be its horizontal and vertical components respectively.
Similarly, let v be the resultant velocity after 5 sec and v_x and v_y be its horizontal and vertical components respectively. β be the angle between v and v_x.

Then, $u = 30ms^{-1}$

$u_x = 30\cos 60° = 15ms^{-1}$

$u_y = 30\sin 60° = 25.98ms^{-1}$

$V = ?$

$V_x = u_x = 15ms^{-1}$

$v_y = u_y - gt$

$= 25.98 - 50$

$= -24ms^{-1}$

Now,

$V = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{15^2 + (-24)^2}$

$= 28.3m^{-1}$

Now,

$\tan\beta = \frac{v_y}{v_x}$

$\tan\beta = \frac{-24}{15}$

$\tan\beta = -1.6$

$\beta = \tan^{-1}(-1.6)$

$\beta = -58°$

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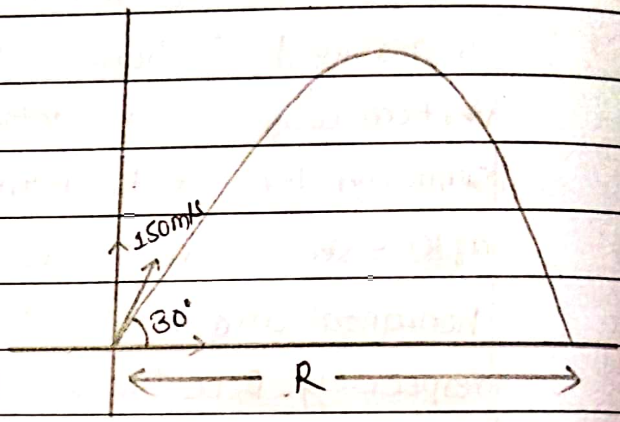
15. A projectile is fired from the ground level with velocity 150m/s at 30° to the horizontal. Find its horizontal range. What will be the least speed with which it can be projected to achieve the same horizontal range? [2068 Old Q.No. 26]

Solution:

Initial velocity (u) = 150m/s

Angle of projection (θ) = 30°

Now,



$$\text{Horizontal range } (R) = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{150^2 \cdot \sin 2(30°)}{10}$$

$$= \frac{22500 \cdot \sin 60}{10}$$

$$= 2250 \times \frac{\sqrt{3}}{2}$$

$$= 1948.55\,m.$$

Again, for the same range, Let, u_{least} be least speed. So,

$$R = \frac{u_{least}^2}{g}$$

$$1948.55 = \frac{u_{least}^2}{10}$$

$$u_{least} = \sqrt{19485.5}$$

$$u_{least} = 140\ m/s.$$

$\therefore$ The least speed with which it can be projected to achieve the same horizontal range is 140m/s.

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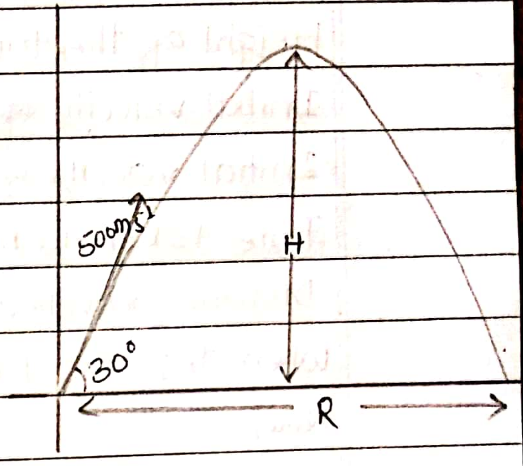
16. A projectile is fired from the ground level with a velocity of $500 ms^{-1}$ at 30° to horizon. Find the horizontal range, and greatest vertical height to which it rises. What is the least speed with which it could be projected in order to achieve the same horizontal range? [$g = 10 N kg^{-1}$]



Solution:

Initial velocity (u) = $500 ms^{-1}$

Angle of projection (θ) = 30°

Now,

Horizontal Range (R) = $\dfrac{u^2 \sin 2\theta}{g}$

$= \dfrac{500^2 \times \sin 60^\circ}{10}$

$= 25000 \times \sqrt{3}/2$

$= 21650.63 m$

Greatest vertical height (H) = $\dfrac{u^2 \sin^2 \theta}{2g}$

$= \dfrac{500^2 \times \sin^2 30^\circ}{2 \times 10}$

$= 12500 \times \dfrac{1}{4}$

$= 3125 m$

Again, for the same horizontal range, let u_{least} be least speed. So,

$R = \dfrac{u_{least}^2}{g}$

$21650.63 = \dfrac{u_{least}^2}{10}$

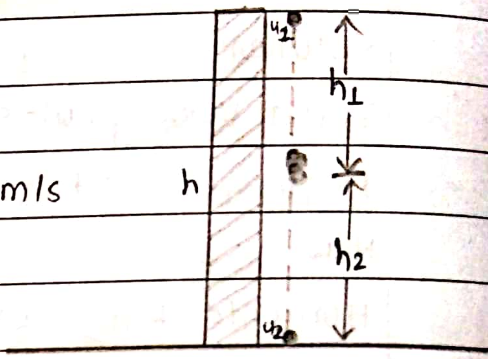
$u_{least} = \sqrt{216506.3}$

$u_{least} = 465.3 m/s$

$\therefore$ The least speed with with it could be projected in order to achieve the same horizontal range is $465.3 m/s$.

17 An object is dropped from the top of the tower of height 156.8m and at the same time another object is thrown vertically upward with the velocity of $78.1 ms^{-1}$ from the foot of the tower. When and where the objects meet? [2063 Q. No. 26]

Solution:



Height of the tower (h) = 156.8m

Initial velocity of first object $(u_1) = 0 m/s$

Initial velocity of second object $(u_2) = 78.1 m/s$

Time taken to meet two objects (t) = ?

Distance travelled by the first object when they meet (h_1) = ?

Now,

For first object

$$h_1 = \frac{1}{2} g t^2 \quad [\because u = 0]$$

For second object,

$$h_2 = ut - \frac{1}{2} g t^2$$

Then, $h_1 + h_2 = \frac{1}{2} g t^2 + ut - \frac{1}{2} g t^2$

or, $156.8 = 78.1 \times t$

or, $t = \frac{156.8}{78.1}$

$\therefore$ $t = 2 sec.$

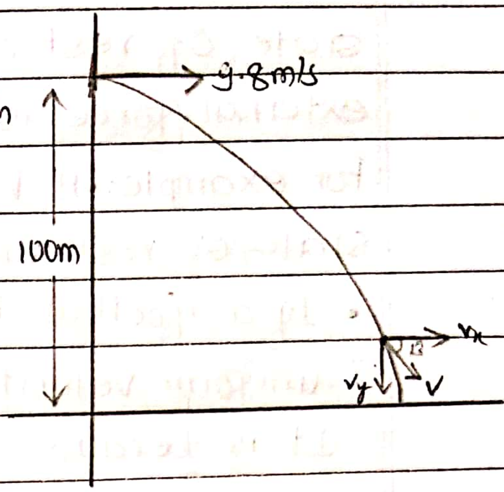
$\therefore$ $h_1 = \frac{1}{2} \times 10 \times 2^2$

$= 20m$

$\therefore$ The objects meet at a distance of 20m from the top of the tower after 2 seconds.

18. A body is projected horizontally from the top of a tower 100m high with a velocity of $9.8ms^{-1}$. Find the velocity with which it hits the ground. [2060 Q. No. 2b]

Solution:



Let, v be the resultant velocity with which the body hits the ground.

Then, Initial velocity (u) = 9.8m/s

Height of the tower (h) = 100m

We know,

Time of flight $(T) = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2 \times 100}{10}}$

$= \sqrt{20}\,s$

Now,

$v_x = u = 9.8m/s$

$v_y = u_y + gt$

$= 10\sqrt{20}$ [$\because u_y = 0$]

$= 44.72$ m/s

$\therefore V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(9.8)^2 + (44.72)^2}$

$= \sqrt{2096.04}$

$= 45.78$ m/s

$\therefore$ The body hits the ground with the velocity of 45.78m/s.